RESEARCH ARTICLE
doi: 10.2306/scienceasia1513-1874.2018.44.129

ScienceAsia 44 (2018): **129–134**

Cycle-supermagic labelling of some classes of plane graphs

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Received 8 Aug 2017
Accepted 26 Nov 2017

ABSTRACT: A simple graph $G = (V, E)$ admits an H-covering if every edge in $E(G)$ belongs to a subgraph of G isomorphic to H. The graph G is said to be H-magic if there exists a bijection $\psi : V(G) \cup E(G) \to \{1, 2, \dots, |V(G)| + |E(G)|\}$ such that for every subgraph H' of G isomorphic to H, the sum $\sum_{v\in V(H')} \psi(v) + \sum_{e\in E(H')} \psi(e)$ is constant. Furthermore, G is said to be H-supermagic if $\psi(V(G)) = \{1, 2, \dots, |V(G)|\}$. In this paper, we study the cycle-supermagic labelling of a pumpkin graph and two classes of planar maps containing 8-sided and 4-sided faces or 6-sided and 4-sided faces, respectively.

KEYWORDS: total labelling, edge covering, H-supermagic labelling

MSC2010: 05C78

INTRODUCTION

An edge covering of G is a family of subgraphs $H_1, \dots, H_r$ such that each edge of $E(G)$ belongs to at least one of the subgraphs H_i, $1 \leqslant i \leqslant r$. Then it is said that G admits an $(H_1, H_2, \dots, H_r)$-(edge)-covering. If every H_i is isomorphic to a given graph H, then G admits an H-covering. Suppose G admits an H-covering. A total labelling $\psi : V(G) \cup E(G) \to \{1, 2, \dots, |V(G)| + |E(G)|\}$ is called an H-magic labelling of G if there exists a positive integer k (called the magic constant) such that for every subgraph H' of G isomorphic to H,

$$\sum_{v\in V(H')} \psi(v) + \sum_{e\in E(H')} \psi(e) = k.$$

A graph that admits such a labelling is called H-magic. An H-magic labelling ψ is called an H-supermagic labelling if $\psi(V(G)) = \{1, 2, \dots, |V(G)|\}$. A graph that admits an H-supermagic labelling is called H-supermagic. The sum of all vertex and edge labels on H (under a labelling ψ) is denoted by $\sum \psi(H)$.

The H-supermagic labelling was first introduced by Gutiérrez and Lladó[1]. They proved that some classes of connected graphs are H-supermagic; e.g., the stars $K_{1,n}$ and the complete bipartite graphs $K_{n,m}$ are $K_{1,h}$-supermagic for some h. They also proved that the paths P_n and the cycles C_n are P_h-supermagic for some h. Jeyanthi and Selvagopal[2] proved that a one point union of n copies of a 2-connected graph G is H-supermagic for any positive integer n. Lladó and Moragas[3] studied some C_n-supermagic graphs. They proved that the wheels W_n, the windmill graphs $W(r, k)$, and the prisms $C_n \times P_2$ are C_h-supermagic for some h. P_h-supermagic labellings of some classes of trees such as the subdivision of stars, shrubs, and banana trees can be found in Ref. 4. Ngurah et al[5] studied cycle-supermagic labellings of chain graphs, fans, triangle ladders, graphs obtained by joining a star $K_{1,n}$ with one isolated vertex, grids, and books. In Ref. 6 it is proved that the disjoint union of an arbitrary number of copies of a C_n-supermagic (C_n-magic) graph is also a C_n-supermagic (C_n-magic) graph. Further results can be found in Refs. 7, 8.

For $H \cong K_2$, an H-supermagic graph is also called a super edge-magic graph. The notion of a super edge-magic graph was introduced in Ref. 9 as a particular type of edge-magic graph given in Ref. 10. The H-magic labelling is related to a face-magic labelling of a plane graph introduced by Lih[11] (see also Refs. 12–14). A labelling of type (1,1,0) (i.e., a total labelling) of a plane graph is said to be face-magic if for every positive integer s, all s-sided faces have the same weight. The weight of a face under a labelling of type (1,1,0) is the sum of labels carried by the edges and vertices surrounding that face. When a plane graph G contains only n-sided faces, then the face-magic labelling of G is also a C_n-magic labelling.

In this paper, we study cycle-supermagic labelling of subdivided graphs and two classes of planar maps containing 8-sided and 4-sided faces or 6-sided and 4-sided faces, respectively.

SUBDIVIDED GRAPHS

Consider the graph $S(G)$ obtained by subdividing some edges of a graph G (and thus inserting some new vertices to the original graph G). Equivalently, the graph $S(G)$ can by obtained from G by replacing some edges of G by paths.

Let G be a graph admitting an H-covering given by t subgraphs $H_1, H_2, \dots, H_t$ isomorphic to H. Consider the subgraphs $S_G(H_i)$, $i = 1, 2, \dots, t$ corresponding to H_i in $S(G)$. If these subgraphs are all isomorphic to a graph (let us denote it by the symbol $S_G(H)$), then the graph $S(G)$ admits an $S_G(H)$-covering.

The next theorem shows that the property of being H-supermagic is hereditary according to the operation of subdivision of edges. This result is a generalization of the results proved in Refs. 15, 16.

Theorem 1 *Let G be an H-supermagic graph and let H_i, $i = 1, 2, \dots, t$ be all subgraphs of G isomorphic to H. If $S_G(H_i)$, $i = 1, 2, \dots, t$ are all subgraphs of $S(G)$ isomorphic to $S_G(H)$ then the graph $S(G)$ is an $S(H)$-supermagic graph.*

Proof: Let G be an H-supermagic graph and let H_i, $i = 1, 2, \dots, t$ be all subgraphs of G isomorphic to H. Let f be an H-supermagic labelling of G. Thus $f : V(G) \cup E(G) \to \{1, 2, \dots, |V(G)| + |E(G)|\}$ such that the vertices of G are labelled with numbers $1, 2, \dots, |V(G)|$ and the weights of subgraphs H_i are

$$\sum_{v \in V(H_i)} f(v) + \sum_{e \in E(H_i)} f(e) = \sum f(H)$$

for every $i = 1, 2, \dots, t$, where $\sum f(H)$ is the magic constant of f.

Consider the graph $S(G)$ obtained from G by inserting p new vertices, say $v_1, v_2, \dots, v_p$. Let $S_G(H_i)$, $i = 1, 2, \dots, t$ be all subgraphs of $S(G)$ isomorphic to $S_G(H)$. Then $S(G)$ admits the $S_G(H)$-covering. Let r denote the number of new vertices inserted into every subgraph $S_G(H_i)$, $i = 1, 2, \dots, t$.

We define a labelling g of $S(G)$ in the following way:

$$g(v) = \begin{cases} f(v), & v \in V(G), \\ |V(G)| + j, & v = v_j,\ j = 1, 2, \dots, p. \end{cases}$$

Evidently, the vertices of $S(G)$ are labelled with distinct numbers $1, 2, \dots, |V(G)| + p$.

Let us choose an orientation of edges in the graph G. According to this orientation we orient the edges in $S(G)$. To an arc uv in G there will correspond the oriented path P_{uv} with initial vertex u and terminal vertex v in $S(G)$. We label the arcs of $S(G)$ such that

$$g(uw) = \begin{cases} f(uv) + p, \\ \quad \text{if } u \in V(G) \text{ and } uw \text{ is an arc on } P_{uv}, \\ |V(G)| + |E(G)| + 2p + 1 - j, \\ \quad \text{if } u = v_j,\ j = 1, 2, \dots, p. \end{cases}$$

The edges are labelled with distinct numbers from the set $|V(G)| + p + 1$, $|V(G)| + p + 2, \dots, |V(G)| + |E(G)| + 2p$. Furthermore, under the labelling g, the weights of subgraphs $S_G(H_i)$, $i = 1, 2, \dots, t$ are

$$\begin{aligned} \mathrm{wt}_g(G(H_i)) &= \sum_{v \in V(G(H_i))} g(v) + \sum_{e \in E(G(H_i))} g(e) \\ &= \sum_{v \in V(H_i)} g(v) + \sum_{v_j \in V(G(H_i))} g(v_j) + \sum_{\substack{e \in E(P(uv)),\\ uv \in E(H_i)}} g(e) \\ &= \sum_{v \in V(H_i)} f(v) + \sum_{v_j \in V(G(H_i))} (|V(G)| + j) + \sum_{e \in E(H_i)} (f(e) + p) \\ &\quad + \sum_{v_j \in V(G(H_i))} (|V(G)| + |E(G)| + 2p + 1 - j) \\ &= \sum_{v \in V(H_i)} f(v) + \sum_{e \in E(H_i)} f(e) + |E(H_i)|p \\ &\quad + (2|V(G)| + |E(G)| + 2p + 1)r \\ &= \sum f(H) + |E(H_i)|p \\ &\quad + (2|V(G)| + |E(G)| + 2p + 1)r. \end{aligned}$$

As $|E(H_i)| = |E(H)|$ for $i = 1, 2, \dots, t$, we obtain that the $S_G(H_i)$ weights are all the same. Thus g is an $S(H)$-supermagic labelling of $S(G)$ with the magic constant $\sum f(H) + |E(H)|p + (2|V(G)| + |E(G)| + 2p + 1)r$. □

Combining Theorem 1 with known results on cycle-magicness of some graphs we immediately obtain new classes of graphs that are cycle-supermagic. Note that it is not necessary to consider only regular subdivisions of graphs.

CYCLE-SUPERMAGIC LABELLING OF SUBDIVIDED WHEELS

A wheel W_n is a graph obtained by joining a single vertex to all vertices of a cycle on n vertices. The vertex of degree n is called the central vertex, or the hub vertex, and the remaining vertices are called the rim vertices. The edges adjacent to the central vertex are called spokes and the remaining edges are called rim edges. Let us denote by the symbol $W_n(r,s)$ the graph obtained by inserting r, $r \geqslant 0$ new vertices into every rim edge and s, $s \geqslant 0$ new vertices into every spoke in the wheel W_n. The graph isomorphic to the subdivided wheel $W_n(r,0)$ is also known as the Jahangir graph $J_{n,r+1}$.

Proposition 1 (Ref. 3) *For odd n, $n \geqslant 5$, the wheel W_n is C_3-supermagic.*

Thus we obtain

Corollary 1 *Let r, s be non-negative integers. For odd n, $n \geqslant 5$, the subdivided wheel $W_n(r,s)$ is C_{3+r+2s}-supermagic.*

Note that this result was proved in Ref. 3. The C_3-supermagicness of wheels was studied in Ref. 17.

Proposition 2 (Ref. 17) *For odd n, $n \geqslant 5$, the wheel W_n is C_4-supermagic.*

Immediately, we obtain

Corollary 2 *Let r, s be non-negative integers. For odd n, $n \geqslant 5$, the subdivided wheel $W_n(r,s)$ is $C_{4+2r+2s}$-supermagic.*

Theorem 2 *The subdivided wheel $W_n(1,0)$, $n \geqslant 3$, is C_{2k+2}-supermagic for $1 \leqslant k \leqslant n-1$.*

Proof: Let us denote vertices and edges of $W_n(1,0)$ as follows:

$$V(W_n(1,0)) = \{c, v_i, u^i : i = 1,2,\dots,n\},$$
$$E(W_n(1,0)) = \{cv_i, v_iu^i, u^iv_{i+1} : i = 1,2,\dots,n\},$$

where the indices are taken modulo n. We define a total labelling $g : V(W_n(1,0)) \cup E(W_n(1,0)) \to \{1,2,\dots,5n+1\}$ in the following way:

$$\begin{aligned} g(c) &= 1, \\ g(v_i) &= 2i, && 1 \leqslant i \leqslant n, \\ g(u^i) &= 2n-2i+3, && 1 \leqslant i \leqslant n, \\ g(cv_i) &= 3n+2-i, && 1 \leqslant i \leqslant n, \\ g(v_iu^i) &= 3n+1+i, && 1 \leqslant i \leqslant n, \\ f(u^iv_{i+1}) &= 5n+1-i, && 1 \leqslant i \leqslant n-1, \\ g(u^nv_1) &= 5n+1. \end{aligned}$$

We denote by the symbol C^i_{2k+2}, $1 \leqslant i \leqslant n$, the $(2k+2)$-cycle such that $C^i_{2k+2} = cv_iu^iv_{i+1}u^{i+1}v_{i+2}\dots u^{i+k-1}v_{i+k}c$, where the index i is taken modulo n. Under the labelling g, the weights of C^i_{2k+2} are as follows:

$$\begin{aligned} \mathrm{wt}_g(C^i_{2k+2}) = g(c) + \sum_{j=0}^{k} g(v_{i+j}) + \sum_{j=0}^{k-1} g(u^{i+j}) \\ + g(cv_i) + g(cv_{i+k}) + \sum_{j=0}^{k-1} g(v_{i+j}u^{i+j}) \\ + \sum_{j=0}^{k-1} g(u^{i+j}v_{i+j+1}). \end{aligned}$$

It is easy to prove that for every i, $1 \leqslant i \leqslant n$, the cycle weights are constant:

$$\mathrm{wt}_g(C^i_{2k+2}) = (10n+6)k + 6n + 5.$$

Hence $W_n(1,0)$ is C_{2k+2}-supermagic. □

Combining Theorem 2 and Theorem 1 we immediately obtain the following result.

Theorem 3 *The subdivided wheel $W_n(r,s)$, $n \geqslant 3$, $r \geqslant 1$ and $s \geqslant 0$ is $C_{k(r+1)+2s}$-supermagic for $1 \leqslant k \leqslant n-1$.*

TWO CLASSES OF PLANAR MAPS

Let n, m be positive integers. Let O^m_n denote a planar map with m rows and n columns of octagons and 4-sided faces between them (Fig. 1). More precisely, the graph O^m_n has the vertex set

$$\begin{aligned} V(O^m_n) = \{a^j_i, b^j_i : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m+1\} \\ \cup \{c^j_i, d^j_i : 1 \leqslant i \leqslant n+1, 1 \leqslant j \leqslant m\} \end{aligned}$$

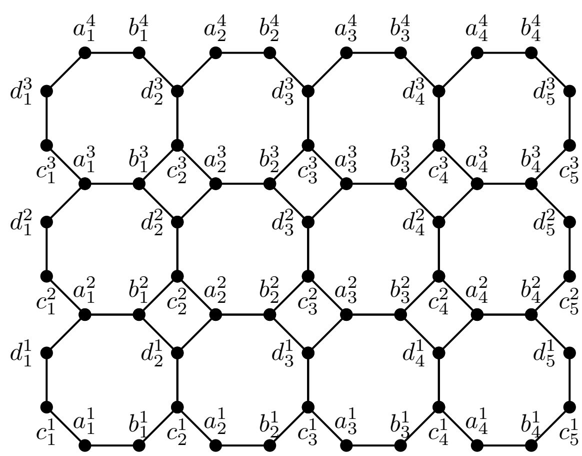

Fig. 1 The planar map O^3_4.

and the edge set

$$E(O_n^m) = \{b_i^{j+1}d_{i+1}^j, a_i^jc_i^j, b_i^jc_{i+1}^j, a_i^{j+1}d_i^j : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m\} \cup \{c_i^jd_i^j : 1 \leqslant i \leqslant n+1, 1 \leqslant j \leqslant m\} \cup \{a_i^jb_i^j : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m+1\}.$$

The map O_n^m is of order $4mn + 2n + 2m$ and of size $6mn + n + m$.

Theorem 4 *Let m, n be positive integers. Then the planar map O_n^m is C_8-supermagic.*

Proof: Define a total labelling $\varphi : V(O_n^m) \cup E(O_n^m) \to \{1, 2, \dots, 10mn + 3n + 3m\}$ as follows. We first label the vertices of the graph O_n^m. For $1 \leqslant j \leqslant m$ and $1 \leqslant i \leqslant n+1$

$$\varphi(c_i^j) = (n+1)(j-1) + i,$$
$$\varphi(d_i^j) = (n+1)(2m-j) + i.$$

For $1 \leqslant j \leqslant m+1$ and $1 \leqslant i \leqslant n$

$$\varphi(a_i^j) = 2m(n+1) + 2n(m+1-j) + 2i - 1,$$
$$\varphi(b_i^j) = 2m(n+1) + 2nj + 2 - 2i.$$

Evidently, the vertices of the graph O_n^m are labelled with distinct numbers from the set $\{1, 2, \dots, 4mn + 2n + 2m\}$.

The edges of O_n^m are labelled such that for $1 \leqslant j \leqslant m$ and $1 \leqslant i \leqslant n$

$$\varphi(b_i^{j+1}d_{i+1}^j) = 2m(2n+1) + 2n + nj + 1 - i,$$
$$\varphi(a_i^jc_i^j) = 2m(3n+1) + 3n - nj + 1 - i,$$
$$\varphi(b_i^jc_{i+1}^j) = 2m(3n+1) + 2n + nj + 1 - i,$$
$$\varphi(a_i^{j+1}d_i^j) = 7mn + 2n + 2m + nj + 1 - i$$

and for $1 \leqslant j \leqslant m+1$ and $1 \leqslant i \leqslant n$

$$\varphi(a_i^jb_i^j) = 3n(3m+1) + 2m - nj + i.$$

To label the edges $c_i^jd_i^j$, $1 \leqslant j \leqslant m$, $1 \leqslant i \leqslant n+1$ we distinguish two cases according to the parity of n. If n is odd then

$$\varphi(c_i^jd_i^j) = \begin{cases} 10mn + 4n + 3m - j(n+1) + 2 - i, \\ \quad i \text{ even}, 2 \leqslant i \leqslant n+1, \\ 9mn + 2m + 3n + (n+1)j + 1 - i, \\ \quad i \text{ odd}, 1 \leqslant i \leqslant n, \end{cases}$$

and if n is even then

$$\varphi(c_i^jd_i^j) = \begin{cases} 10mn + 4n + 3m - j(n+1) + 2 - i, \\ \quad i \text{ odd}, 1 \leqslant i \leqslant n+1, \\ 9mn + 2m + 3n + (n+1)j + 1 - i, \\ \quad i \text{ even}, 2 \leqslant i \leqslant n. \end{cases}$$

It is easy to see that the edges are labelled with distinct numbers from the set $\{4mn + 2n + 2m + 1, 4mn + 2n + 2m + 2, \dots, 10mn + 3n + 3m\}$.

The planar map O_n^m admits a C_8-covering given by mn cycles on 8 vertices. We denote these cycles by $C_{8,i}^j$, for $1 \leqslant j \leqslant m$, $1 \leqslant i \leqslant n$, with the following vertex sets and edge sets.

$$V(C_{8,i}^j) = \{a_i^j, c_i^j, d_i^j, a_i^{j+1}, b_i^{j+1}, d_{i+1}^j, c_{i+1}^j, b_i^j : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m\},$$
$$E(C_{8,i}^j) = \{a_i^jc_i^j, c_i^jd_i^j, d_i^ja_i^{j+1}, b_i^{j+1}d_{i+1}^j, d_{i+1}^jc_{i+1}^j, b_i^jc_{i+1}^j, a_i^jb_i^j, a_i^{j+1}b_i^{j+1} : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m\}.$$

It is not difficult to verify by a routine procedure that the supermagic constant under the labelling φ, for every $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant m$, has the following value:

$$\sum \varphi(C_{8,i}^j) = \varphi(a_i^j) + \varphi(c_i^j) + \varphi(d_i^j) + \varphi(a_i^{j+1}) + \varphi(b_i^{j+1}) + \varphi(d_{i+1}^j) + \varphi(c_{i+1}^j) + \varphi(b_i^j) + \varphi(a_i^jc_i^j) + \varphi(c_i^jd_i^j) + \varphi(d_i^ja_i^{j+1}) + \varphi(a_i^jb_i^j) + \varphi(b_i^{j+1}d_{i+1}^j) + \varphi(d_{i+1}^jc_{i+1}^j) + \varphi(b_i^jc_{i+1}^j) + \varphi(a_i^{j+1}b_i^{j+1}) = 76mn + 29m + 23n + 8.$$

□

Fig. 2 depicts C_8-supermagic labellings of O_3^2.

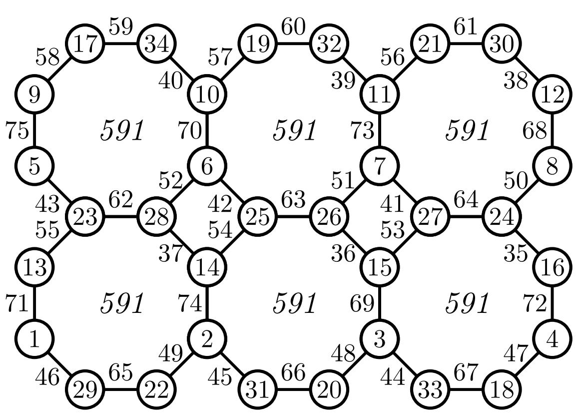

Fig. 2 C_8-supermagic labellings of O_3^2.

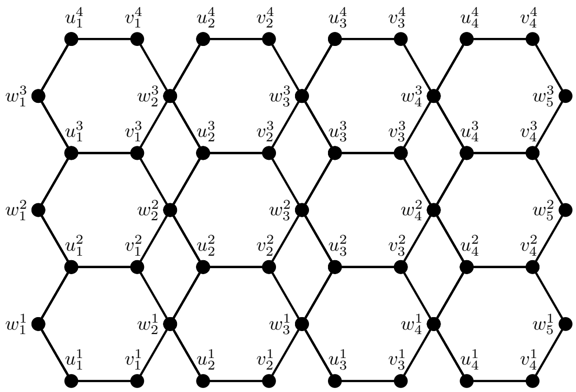

Fig. 3 The planar map H_4^3.

We now consider a planar map H_n^m with $n, m \geqslant 1$ illustrated in Fig. 3 with m rows and n columns of hexagons and 4-sided faces between them. Let

$$\begin{aligned} V(H_n^m) &= \{u_i^j, v_i^j : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m+1\} \\ &\quad \cup \{w_i^j : 1 \leqslant i \leqslant n+1, 1 \leqslant j \leqslant m\} \\ E(H_n^m) &= \{v_i^{j+1} w_{i+1}^j, u_i^j w_i^j, v_i^j w_{i+1}^j, w_i^j u_i^{j+1} : \\ &\quad 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m\} \\ &\quad \cup \{u_i^j v_i^j : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m+1\} \end{aligned}$$

be the vertex set and the edge set of H_n^m, respectively. Thus $|V(H_n^m)| = 3mn+2n+m$ and $|E(H_n^m)| = 5mn+n$.

Theorem 5 *Let m, n be positive integers. Then the planar map H_n^m is C_6-supermagic.*

Proof: We define a total labelling $\varphi : V(H_n^m) \cup E(H_n^m) \to \{1, 2, \dots, 8mn+3n+m\}$ in the following way. For $1 \leqslant j \leqslant m+1$ and $1 \leqslant i \leqslant n$

$$\begin{aligned} \varphi(u_i^j) &= 2n(j-1)+2i-1, \\ \varphi(v_i^j) &= 2n(m+2-j)+2-2i, \\ \varphi(u_i^j v_i^j) &= 8mn+3n+m-jn+i. \end{aligned}$$

For $1 \leqslant j \leqslant m$ and $1 \leqslant i \leqslant n+1$

$$\varphi(w_i^j) = \begin{cases} 2n(m+1)+(n+1)(m-j)+i, & i \text{ odd}, \\ 2n(m+1)+(n+1)(j-1)+i, & i \text{ even}. \end{cases}$$

For $1 \leqslant j \leqslant m$ and $1 \leqslant i \leqslant n$

$$\begin{aligned} \varphi(v_i^{j+1} w_{i+1}^j) &= 3mn+2n+m+jn+1-i, \\ \varphi(u_i^j w_i^j) &= 5mn+3n+m-jn+1-i, \\ \varphi(v_i^j w_{i+1}^j) &= 5mn+2n+m+jn+1-i, \\ \varphi(w_i^j u_i^{j+1}) &= 6mn+2n+m+jn+1-i. \end{aligned}$$

It is easy to check that the labelling φ is a bijection and that the vertices are labelled with the numbers $1, 2, \dots, 3mn+2n+m$.

The planar map H_n^m admits a C_6-covering and is covered by the cycles $C_{6,i}^j$, for $1 \leqslant j \leqslant m$, $1 \leqslant i \leqslant n$, with the following vertex sets and edge sets:

$$\begin{aligned} V(C_{6,i}^j) &= \{u_i^j, w_i^j, u_i^{j+1}, v_i^{j+1}, w_{i+1}^j, v_i^j : 1 \leqslant i \leqslant n, \\ &\qquad 1 \leqslant j \leqslant m\}, \\ E(C_{6,i}^j) &= \{u_i^j w_i^j, w_i^j u_i^{j+1}, u_i^{j+1} v_i^{j+1}, v_i^{j+1} w_{i+1}^j, \\ &\qquad w_{i+1}^j v_i^j, v_i^j u_i^j : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m\}. \end{aligned}$$

We observe that under the labelling φ, for every $1 \leqslant i \leqslant n$ and $1 \leqslant j \leqslant m$, the supermagic constant attains the value

$$\begin{aligned} \sum \varphi(C_{6,i}^j) &= \varphi(u_i^j) + \varphi(w_i^j) + \varphi(u_i^{j+1}) \\ &\quad + \varphi(v_i^{j+1}) + \varphi(w_{i+1}^j) + \varphi(v_i^j) + \varphi(u_i^j w_i^j) \\ &\quad + \varphi(w_i^j u_i^{j+1}) + \varphi(u_i^{j+1} v_i^{j+1}) + \varphi(v_i^{j+1} w_{i+1}^j) \\ &\quad + \varphi(w_{i+1}^j v_i^j) + \varphi(u_i^j v_i^j) \\ &= 44mn+7m+21n+6. \end{aligned}$$

□

Fig. 4 gives a C_6-supermagic labelling of H_3^2.

CONCLUSIONS

We have examined the existence of cycle-supermagic labelling for subdivided graphs. We proved that the property of being cycle-supermagic is hereditary under the operation subdivision of edges. We showed that the subdivided wheel $W_n(r,s)$ admits cycle-supermagic labellings for all cycles $C_{k(r+1)+2s}$ for $1 \leqslant k \leqslant n-1$, $n \geqslant 3$, $r \geqslant 1$ and $s \geqslant 0$. Furthermore, we examined the existence of C_8-supermagic and C_6-supermagic labellings of

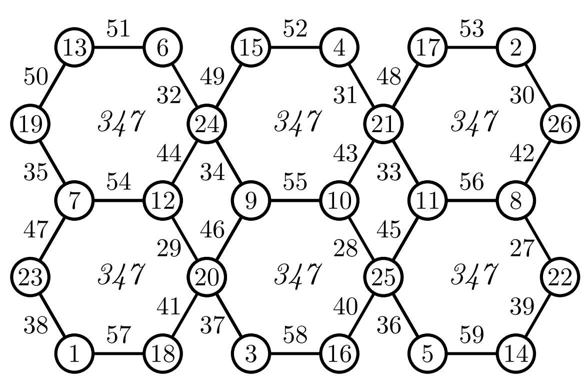

Fig. 4 C_6-supermagic labelling of H_3^2.

planar maps O_n^m and H_n^m. We suggest the following open problem.

Problem 1 *Prove that the subdivided wheel $W_n(0,s)$ is C_{k+2s+2}-supermagic for $1 \leqslant k \leqslant n-1$, $n \geqslant 3$.*

Acknowledgements: The research for this article was supported by APVV-15-0116 and by VEGA 1/0233/18.

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