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RATIO $2\Delta_0/kT_C$ IN HIGH TEMPERATURE SUPERCONDUCTORS

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ABSTRACT

Within the context of weak-coupling BCS theory, we evaluate the ratio $R=2\Delta_0/T_c$ (Δ_0 is zero-temperature energy gap, T_c is critical temperature). By assuming the singular density of states of the form $A|\varepsilon|^{\alpha}+B$, where A, B and α are model parameters and taking correct cut-off limits, we find larger values for R than the recent results of Mattis and Molina, but not large enough to explain the observed experimental results.

INTRODUCTION

The effects of the electronic density of states (DOS) near the Fermi level on superconductivity have attracted much attention recently.[1,2] According to the conventional theory of superconductivity, only the electrons with energies within a range of the order of $\hbar\omega_D$ near the Fermi energy E_F are important. Here ω_D is the Debye frequency for phonon-mediated superconductivity. The BCS theory takes the DOS near the Fermi level to be a constant, this assumption is reasonable in most low-T_c three dimensional superconductors. However, in quasi two-dimensional systems there is a logarithmic van Hove singularity (VHS) in the DOS. Tsuei *et al.*[3] have shown that VHS leads to an enhancement in T_c and a small isotope exponent. Nevertheless, we[4] found recently that within the VHS scenario, the gap ratio R is too small to explain the experimentally indicated values up to 8.0.[5]

The question arises if there is a possibility to extend the value of the gap ratio well above the BCS value of 3.53, taking into account of any DOS singularity. Mattis and Molina[6] evaluated the zero-temperature-gap-ratio with the singular density of states of the form $N(\varepsilon) = A|\varepsilon|^{\alpha}$, have $\varepsilon = E_k-E_F$ and found a slow decrease of the value R from $R = 4$ at $\alpha = -0.8$ to a low $R = 2.9$ at $\alpha = 1$. Abrikosov *et al.*,[7] using the observed extended saddle point singularities along the Γ-Y symmetry direction in a 1-2-3 high-T_c superconductor showed that the DOS diverges as the square root of energy. Using the Eliashberg theory and a model DOS of the form $N(\varepsilon) = B|\varepsilon|^{-1/4}$, Zeyhar[8] showed that large enhancements of T_c and strong reductions of the isotope exponent cannot be explained.

On examining the BCS-based work of ref. 6, we found that the upper limit of integration was taken to be infinite. In view of this inaptness, we would like to make a contribution by readdressing the issue of the gap behaviour.

The zero-temperature energy gap Δ_0 is obtained by solving the equation

$$\frac{2}{V} = \int_{E_F-\omega_D}^{E_F+\omega_D} \frac{N(\varepsilon)}{\sqrt{\varepsilon^2+\Delta_0^2}} d\varepsilon \tag{1}$$

for a cut-off frequency ω_D, constant pairing potential V.

The critical temperature T_c is obtained from the linearized BCS equation

$$\frac{2}{V} = \int_{E_F-\omega_D}^{E_F+\omega_D} \frac{N(\varepsilon)}{\varepsilon} tanh\left(\frac{\varepsilon}{2T_c}\right) d\varepsilon \tag{2}$$

We take the DOS of the form

$$N(\varepsilon)=A|\varepsilon|^{\alpha}+B \qquad (3)$$

where B is the background DOS which must be included, according to tight-binding calculations for a two-dimensional lattice.[9]

From equations (1)-(3), we obtain the exact formula for R

$$R=4\left[\frac{\int_0^{\omega_D/2T_c} dx\left[x^{\alpha}+\frac{B/A}{(2T_c)^{\alpha}}\right]\frac{\tanh x}{x}}{\int_0^{2\omega_D/RT_c} dx\left[x^{\alpha}+\frac{B/A}{(RT_c/2)^{\alpha}}\right]\frac{1}{\sqrt{1+x^2}}}\right]^{\frac{1}{\alpha}} \qquad (4)$$

for $\alpha \le 0$, and for $\alpha > 0$, we have

$$R=4\left[\frac{\int_0^{\omega_D/2T_c} dx\left[x^{\alpha}+\frac{B/A}{(2T_c)^{\alpha}}\right]\frac{[1-\tanh x]}{x}}{\int_0^{2\omega_D/RT_c} dx\frac{\left[x^{\alpha}+\frac{B/A}{(RT_c/2)^{\alpha}}\right]}{x}\left[1-\frac{1}{\sqrt{1+\frac{1}{x^2}}}\right]}\right]^{\frac{1}{\alpha}} \qquad (5)$$

Our equations (4) and (5) reduce to equations (3) and (4) of ref. 6 in the $B=0$ limit.

A numerical calculation of R based on equations (4) and (5) with $B=0$ is shown in Fig. 1 as a function of α for different values of ω_D/T_c. It can be seen from the graph that, with the proper range of integration, our calculation yields higher values R than the ref. 6's results. We conclude that the finite limits of integration always increase R, but not sufficiently to explain the experimentally observed high value of R. We also observed that the choice of larger ω_D/T_c does lead to a decrease in R, this is because it enlarges the effective region of the DOS.

When $B\neq 0$, we compute numerically the gap ratio R from equations (4) and (5) as a function of the parameter α for different values of B/A using $\omega_D=400\,K$ and $T_c=90\,K$. It can be seen in Fig. 2 that R varies from about 4 down to about 3.4 as α is varied from -0.9 to $+0.9$. The general behaviour of R is that it is larger for $\alpha<0$ than for than $\alpha>0$. As α becomes more and more negative, we find that R increases as $B/A\to 0$, and as B/A increases, R tends to value $R(0)$. We note that the opposite is true when α becomes more and more positive R decreases as $B/A\to 0$.

In conclusion, we have solved the gap-ratio numerically for a high temperature superconductor using the density of states of the form $N(\varepsilon)=A|\varepsilon|^{\alpha}+B$. We find that R depends sensitively on the material parameters. By keeping T_c and ω_D fixed, we note that for $\alpha<0$, R increases as $B/A\to 0$ and for $\alpha>0$, R decreases as $B/A\to 0$. The overall behaviour of the gap-ratio show unambiguously that it still cannot explain the anomalously high values of R as observed in ref. 5.

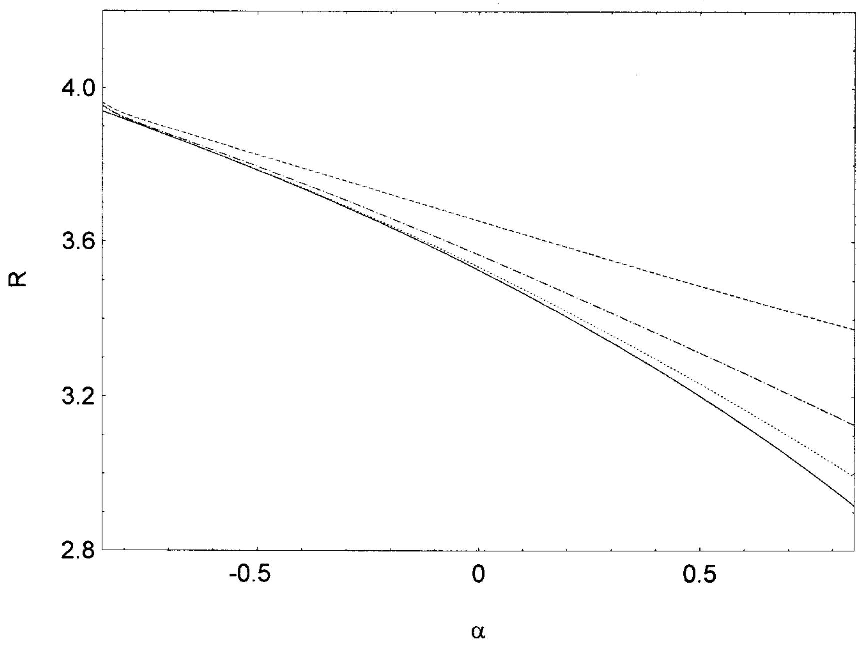

Fig. 1 Plot of ratio $R=2\Delta_0/kT_c$ for different choices of ω_D/T_c and $B=0$ as a function of the exponent α. Bottom curve is Mattis and Molina's result, $\omega_D/T_c \rightarrow \infty$(——), ω_D/T_c = 754/90(-·-·-·-), ω_D/T_c = 754/40(........) and ω_D/T_c = 400/90(- - - -)

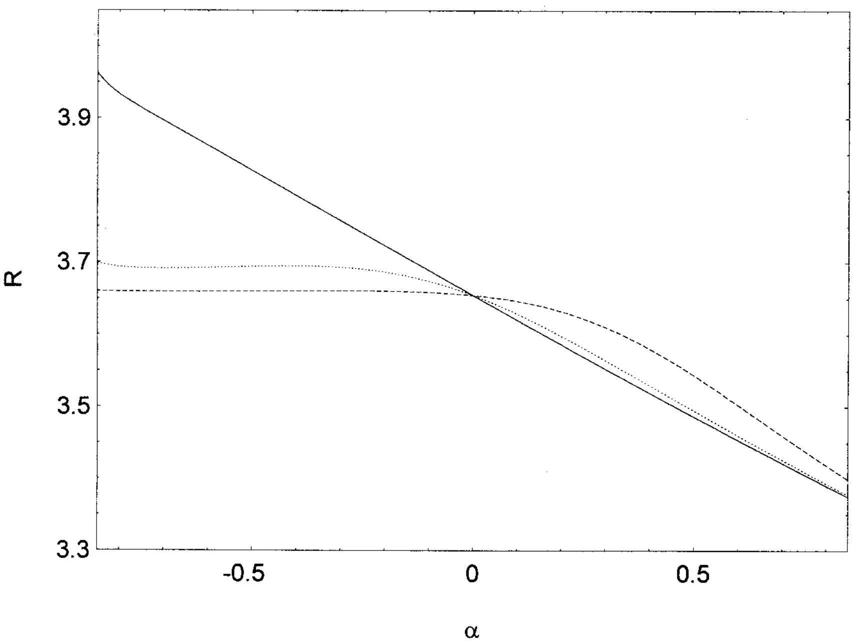

Fig. 2 Values of R as a function of α for different B/A values. We take ω_D=400 K and T_c=90 K, B/A = 0.0(——), B/A = 0.5(........) and B/A = 5.0(- - - -)

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REFERENCES

1. Labbe, J. and Bok, J. (1987) ***Europhys. Lett.* 3**, 1225.
2. Newns, D. M., Pattnaik, P. C. and Tsuei, C. C. (1991) ***Phys. Rev. B* 43**, 3075.
3. Newns, D. M., Pattnaik, P. C. and Tsuei, C. C. and Kane, C. L. (1992) ***Phys. Rev. Lett.* 69**, 2134.
4. Ratanaburi, S., Udomsamuthirun, P. and Yoksan, S. ***Journal of Superconductivity*** (in press).
5. Wolf, S. A. and Krezin, V. Z. (1991) ***IEEE, Tran. Mag.* 21**, 852.
6. Mattis, D. C. and Molina, M. (1991) ***Phys. Rev. B* 44**, 12565.
7. Abrikosov, A. A., Campuzano, J. C. and Gofron, K. (1993) ***Physica C* 412**, 73.
8. Zeyhar, R. (1995) ***Z. Phys.* 97**, 3.
9. Xing, D. Y., Lui, M., Gong, C. D. (1991) ***Phys. Rev. B* 44**, 12525.