J.Sci.Soc.Thailand, 22 (1996) 53-60

COMPUTER SIMULATION OF ELECTROMAGNETIC WAVE PROPAGATION THROUGH UNDERSIZED WAVE GUIDES

KLAUS HAβ

Department of Physics, Faculty of Science, Ubon Ratchathani University, Ubon Ratchathani, 34190, Thailand.

(Received December 26,1995)

ABSTRACT

The propagation of electromagnetic wave packets through undersized wave guides is not yet clearly understood. Microwave experiments carried out by Nimtz and Enders [1] showed, that under special circumstances, the propagation time through an undersized wave guide is independent of its length. Thus superluminal signal transport cannot be ruled out in principle. Nevertheless it was shown [2], that these results are in accordance with Einstein causality, and strictly time bounded signals propagate not faster than light. Here the propagation of different kinds of wave packets is simulated, and it clearly can be seen, that superluminal signal transport is possible under special circumstances.

1. INTRODUCTION

The Helmholtz wave equation and the Schrödinger equation have the same structure with harmonical time dependence. Furthermore the comparison between the tunnelling of a quantum mechanical wave packet through a constant potential barrier and an electromagnetic wave packet propagating through a wave guide with rectangular cross-section leads to analogous boundary conditions. Differences between these two systems appear, when the dispersion relation is taken into account.

In both systems the wave vector has a branch point at the so called "cut-off" frequency, and it takes purely imaginary values with frequencies below this frequency. The propagation of wave packets that consists mainly of frequencies below cut-off is not yet clearly understood. There are no oscillations, but exponentially damping of the fields inside the barrier.

Several theoretical investigations [3], [4], [5] led to the result, that superluminal transport of matter waves tunnelling through a potential barrier or electromagnetic wave packets propagating through undersized wave guides is possible under special circumstances. But all investigations had to make special assumptions to the physical systems which led to the question about the possibility of measurement of these velocities.

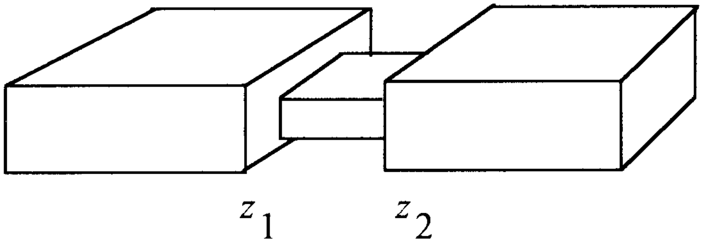
Microwave experiments carried out by Enders and Nimtz [1] led to the result, that the transmission time of these wave packets is independent of the barrier length. Thus superluminal transport cannot be ruled out in principle. Nevertheless it was shown [2], that these results are in accordance with Einstein causality. Similar experiments with photon pairs [6] led to equivalent results.

The frequency-dependent transmission coefficient for electromagnetic waves travelling through wave guides with a rectangular shape is well-known [4], but analytical description of wave packets propagating through these system is not possible, because integration over the frequency range cannot be carried out. Thus computer simulation according to the experiments of Nimtz and Enders for demonstrating the appearance of a wave packet at significant points in space and time was done by the author, using nothing more than the solutions of the wave equation of the system.

2. THEORETICAL FOUNDATIONS

The following investigations are carried out for an electromagnetic wave packet propagating through a wave guide that includes an undersized piece, the barrier. It is in accordance with the experiments of Nimtz and Enders. The formalism for the tunnelling of quantum mechanical matter waves is strictly analogous [5].

Consider a wave guide with cross-section *axb* containing the barrier with cross-section *a'xb'* between z_1 and z_2



The longitudinal parts of a wave packet, that propagates from the left to the right through the device leads to the following Ansatz

$$\psi(z)=\begin{cases} Ae^{ikz}+B_I e^{-ikz} & (z<z_1) \\ A_{II}e^{-\kappa z}+B_{II}e^{\kappa z} & (z_1<z<z_2) \\ A_{III}e^{ikz} & (z>z_2) \end{cases} \tag{1}$$

The time dependent factor $e^{-i\omega t}$ has been neglected here, k and κ are defined by [7]

$$k(\omega)=\begin{cases} \frac{1}{c}\sqrt{\omega^2-\omega_{cI}^2} & (\omega>\omega_{cI}) \\ i\frac{1}{c}\sqrt{\omega_{cI}^2-\omega^2} & (\omega<\omega_{cI}) \end{cases} \tag{2}$$

$$\kappa(\omega)=\begin{cases}\frac{1}{c}\sqrt{\omega_{cII}^2-\omega^2} & (\omega<\omega_{cII})\\ -i\frac{1}{c}\sqrt{\omega^2-\omega_{cII}^2} & (\omega>\omega_{cII})\end{cases} \tag{3}$$

If $b>a$ and $b'>a'$ is assumed, the lowest cut-off frequencies corresponding to a TE_{10} mode are defined by

$$\omega_{cI}=\frac{c\pi}{b} \qquad \omega_{cII}=\frac{c\pi}{b'} \tag{4}$$

The coefficients B_I, A_{II}, B_{II} and A_{III} are uniquely determined by the continuity conditions at the transition points z_1 and z_2 and can be calculated analytically [4]. The transmission coefficient $t(\omega)=\frac{A_{III}}{A}$ is of interest for the further investigation. It is given by

$$t(\omega)=\frac{2e^{-ik(z_2-z_1)}}{2\cosh[\kappa(z_2-z_1)]+\left(\frac{\kappa}{k}-\frac{k}{\kappa}\right)\sinh[\kappa(z_2-z_1)]} \tag{5}$$

Investigation of a gaussian shaped wave packet mainly consisting of frequency components lying between the two cut-off frequencies $\omega_{cI}<\omega<\omega_{cII}$ leads to the result, that the wave packet propagates with the group velocity

$$\frac{d\omega}{dk}=c\sqrt{1-\frac{\omega_{cI}^2}{\omega^2}} \tag{6}$$

through the larger part of the wave guide, and no time is needed to cross the barrier. This effect is mainly caused by the phase factor $e^{-ik(z_2-z_1)}$ of equation (5). Instead of this, the wave packet is modified by the barrier in the way that the maximum of the gaussian function ω_0 is shifted to a higher frequency by the value

$$\Delta\omega = \frac{\omega_o \sigma^2 (z_2 - z_1)}{2c^2 \kappa(\omega_o)} \qquad (7)$$

In this first order approximation, the variance σ of the wave packet remains unchanged [2].

3. COMPUTATIONAL METHODS

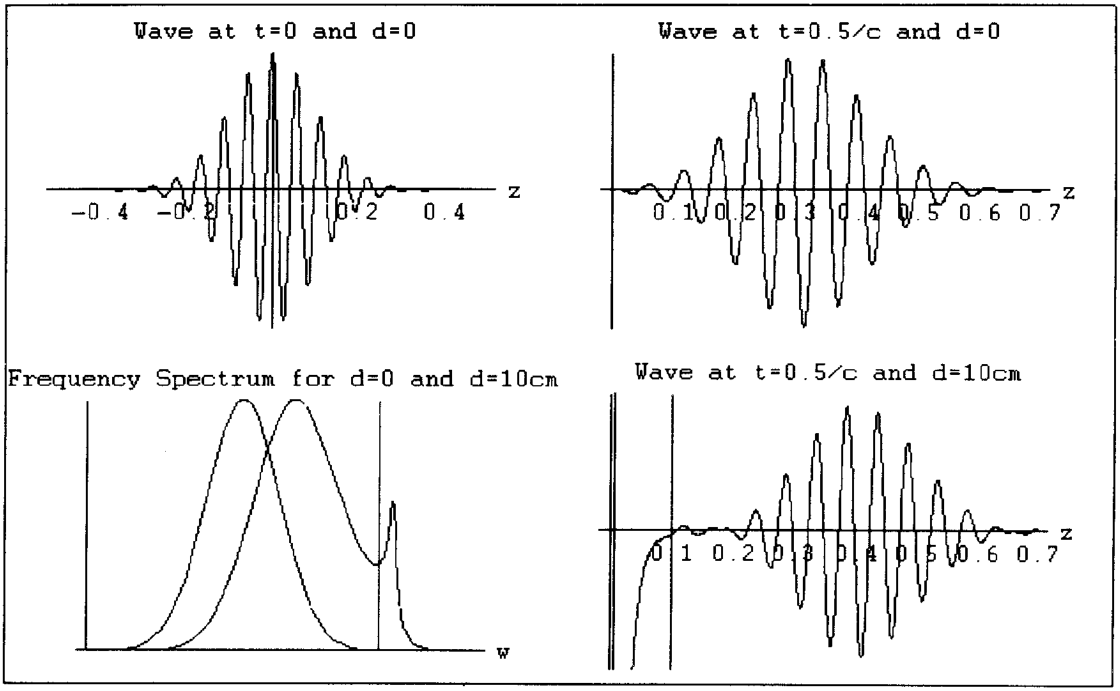
To describe a wave packet as a function in space and time, equation (1) must be integrated over the frequency range of the packet. Because analytical integration cannot be carried out, numerical approximation by computer implementation was made. The frequency range of the wave packet was divided into 100 equally distant intervals and the frequency spectrum was assumed to be constant in these intervals. By summation over this frequency range, the wave packet was calculated and plotted at significant points in space and time. To test the accuracy of the implementation, some plots were generated by dividing the frequency range into 500 intervals. These plots had exactly the same shape, so the length of the intervals was assumed to be small enough, and a division of the frequency range into narrower intervals would only have raised the calculation time. The 100 frequency values according to the intervals were written into a memory to reduce the calculation time.

First a gaussian shaped wave packet was investigated to check the accordance with the analytical results of Busch and Haβ. The transmission properties of the gaussian shaped wave packet does not establish a non-causal behaviour. The packet is not time bounded, and the packet behind the barrier consists of different frequency components compared to the initial packet, so no direct causal connection between the packets are given. For investigation of the causal behaviour, time bounded pulses must be investigated. Furthermore the conditions have to be changed in the way, that the wave packet propagates with the vacuum speed of light outside the barrier. This was simply done by setting the lower cut-off frequency to zero. To clarify the results, two different kinds of single-peaked pulses have been investigated, one pulse consisting mainly of frequencies below cut-off and the other one mainly consisting of frequencies above cut-off. Opposite to the gaussian shaped wave packet, the calculation was based on the time-range of the pulse. So Fourier-transform had to be carried out first to evaluate the frequency spectrum of the pulse.

All numerical values printed in the plots are SI-units. The tunnelling regions are always placed between the y-axes and the grid lines. The barrier length is named by d. With the plots of the frequency spectrum, the grid lines constitute the cut-off frequency of the tunnelling region.

4. RESULTS

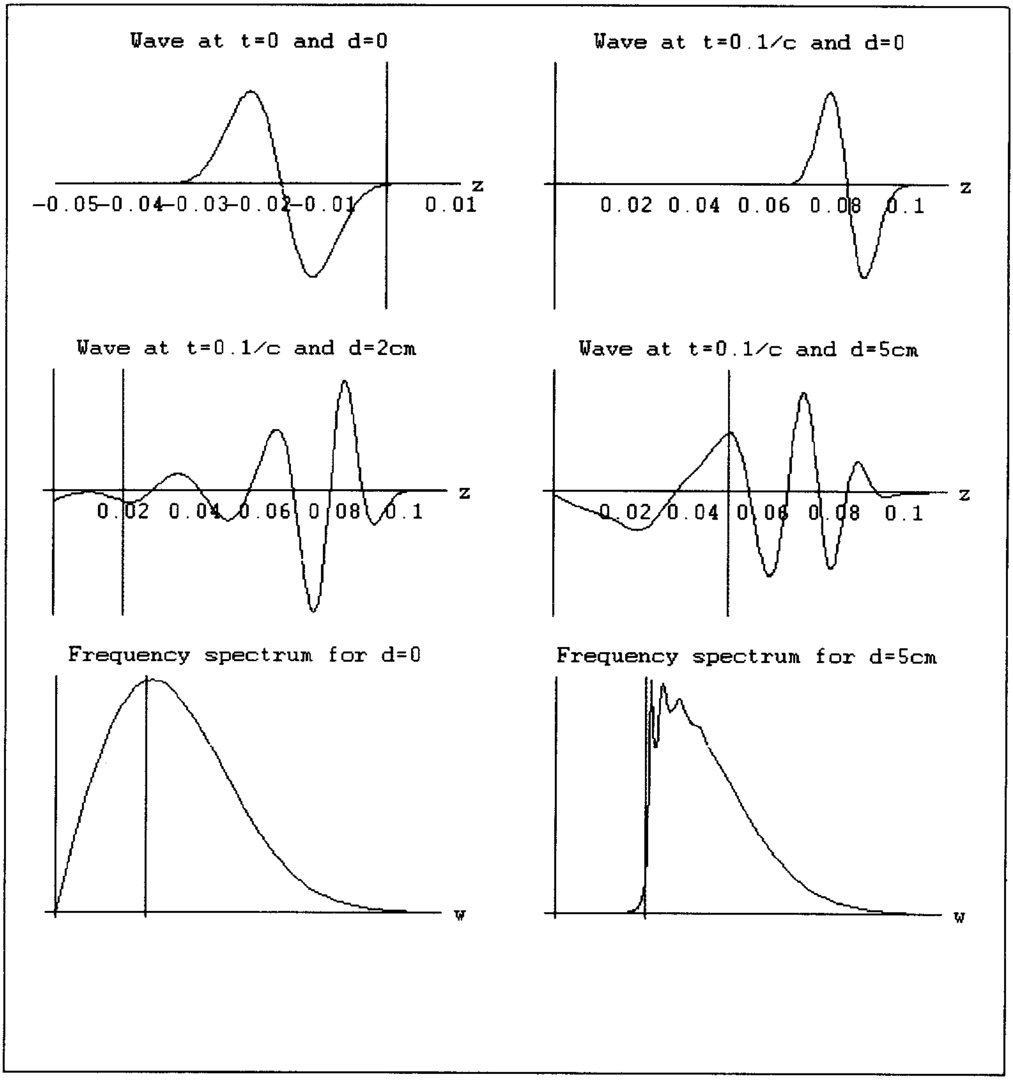
The evaluations of the gaussian shaped wave packets are in accordance with the experiments of Nimtz and Enders and the analytical calculation of Busch and Haβ. Thus the cut-off frequencies, the carrier frequency and the variance are given by: $\omega_{cI}=4.12\ 10^{10}s^{-1}$, $\omega_{cII}=6\ 10^{10}s^{-1}$, $\omega_o=5.3\ 10^{10}s^{-1}$, $\sigma=0.05\omega_o$. The appearance of the wave packet at $t=0$, $t=0.5/c$ for no barrier and for a barrier of 10*cm* thickness and a comparison of the frequency spectra of the incident packet and the tunnelled packet are displayed below.



Gaussian shaped wave packet

A comparison of the wave packets shows, that the tunnelled packet appears indeed 10*cm* in front of the packet that did not cross a barrier. Evaluation of equation (6) leads to a group velocity of $1.9\ 10^8$ m/s at the carrier frequency and is in accordance with the plotted packets. The shape of the wave packet seams to be unchanged, but a comparison of the frequency spectra shows, that the frequency maximum changes to a higher value. Evaluation of equation (7) leads to a frequency shift of $\Delta\omega=2.2\ 10^9\ s^{-1}$ and is in accordance with the plot. It also can be seen, that the transmitted wave packet has frequency components above cut-off, while the components of the initial wave packet in this range can be neglected. Both effects are caused by the high sensitivity of the transmission coefficient concerning ω.

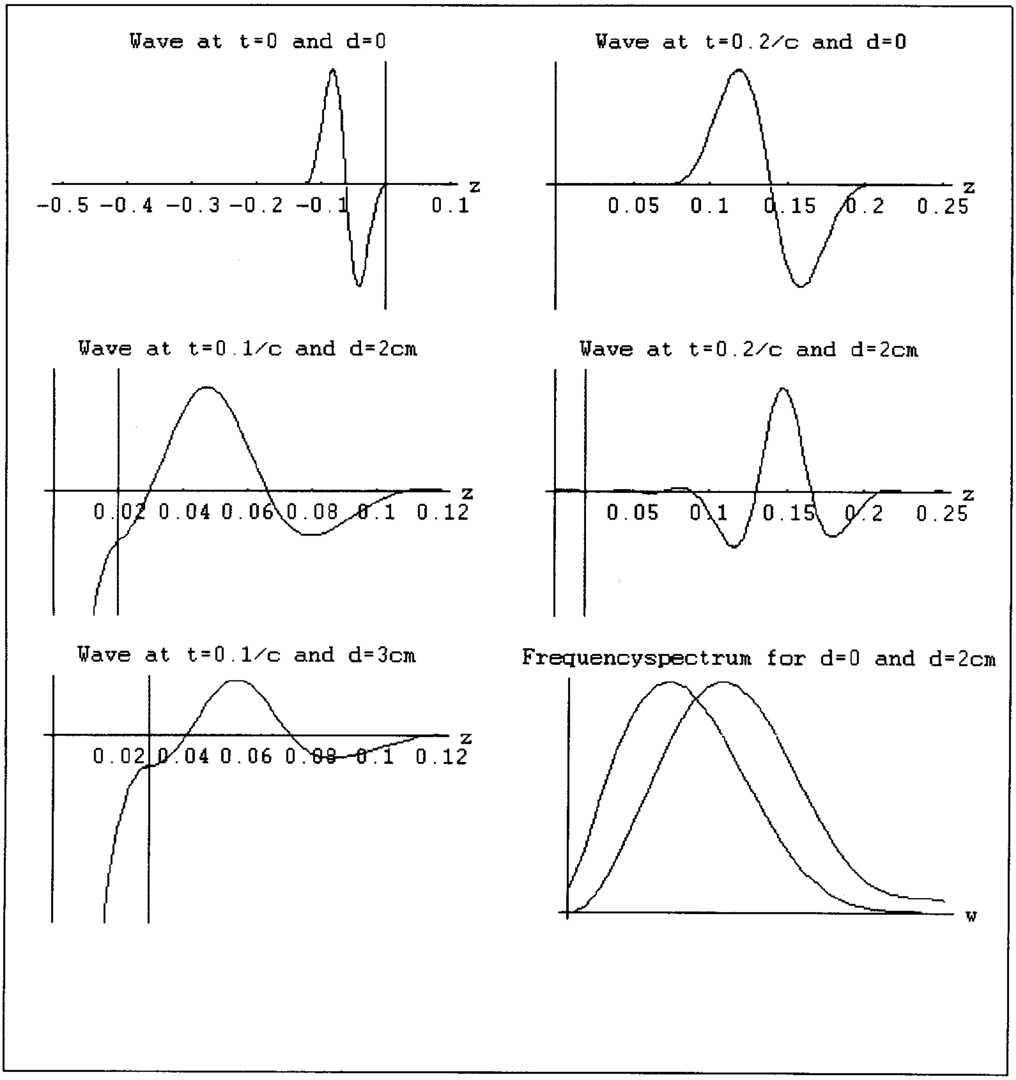
The generation of the time bounded pulse that mainly consists of frequencies above cut-off led to the plots displayed below.



Time bounded pulse consisting mainly of frequencies above cut-off

From the above pictures it can be seen, that the pulse propagates in accordance with Einstein causality as expected and decreases to zero exactly at z=0.1/c, according to the distance that light would have travelled in vacuum. Furthermore these pictures establish, that the pulse becomes modified by the barrier, and the shape changes considerable. Because the pulse consists mainly of frequencies above cut-off, oscillations inside the barrier appear, and the undersized wave guide does not act as a barrier at all. From the plots of the frequency spectra it can be seen, that the frequency components of the tunnelled pulse below cut-off may be neglected, while the incident pulse consists of frequency components above and below cut-off with about the same magnitude.

Examination of a pulse consisting mainly of frequencies below cut-off leads to a different result. From the plots below it can be seen, that the pulse propagates indeed faster than light, and its behaviour is not in accordance with Einstein causality. A comparison of the frequency spectra of the incident pulse and the pulse that tunnelled 2cm are plotted between 0 and the cut-off frequency and it can be seen, that the frequency spectrum of the tunnelled pulse shifts to higher frequencies, but nevertheless the components above cut-off may be neglected. Also the modification of the pulse can be seen clearly and seems to be similar to the modification of the pulse shown above.



Time bounded pulse consisting mainly of frequencies below cut-off

5. DISCUSSION

The above results indeed show, that superluminal signal transport could be possible. Furthermore nothing more than the solution of the wave equation (1) was used, and no assumptions had to be made to the system. Thus faster than light velocities could be measurable in principle, if just the detector sensitivity was high enough. The simulation establishes accordance with the experimental results of Nimtz and Enders and Steinberg *et al.* Nevertheless the contradiction between these results and the analytical calculations of Busch and Hafl is still an open question.

REFERENCES

1. A. Enders and G. Nimtz: Photonic tunnelling experiments. *Physical Review* **B47** (1993), On superluminal barrier traversal. *Journal de Physique* **2**, 1693 (1992), Zero-time-tunnelling of evanescent mode packets, *Journal de Physique* **3**, 1089-1092 (1993)
2. Klaus Hafl and Paul Busch: Causality of superluminal barrier traversal, *Physics Letters* **A185**, 9-13 (1994)
3. S. Bosanac: Propagation of electromagnetic wave packets in nondispersive dielectric media. *Physical Review* **A28 Nr.2**, 577-591 (1983)
4. Th. Martin and L. Landauer: Time delay of evanescent electromagnetic waves and the analogy to particle tunnelling, *Physical Review* **A45**, 2611-2617 1992)
5. Th. E. Hartman: Tunnelling of a wave packet, *Journal of applied physics* **33**, 3427-3433 (1962)
6. A. M. Steinberg, P. G. Kwiat and R. Y. Chiao: Measurement of the single photon tunnelling time. *Physical Review Letters* **71**, 708 (1993)
7. J. D. Jackson Classical Electrodynamics, John Wiley and Sons, New York (1962)