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CHARACTERIZATION OF ELECTRO-OPTIC DIRECTIONAL COUPLER BASED DEVICES

T. WONGCHAROEN, B.M.A. RAHMAN AND K.T.V. GRATTAN

City University, Department of Electrical, Electronic and Information Engineering, Northampton Square, London EC1V 0HB, U.K.

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ABSTRACT

Results are presented on a study of the important parameters of synchronous and nonsynchronous, weakly and strongly coupled optical directional couplers using the finite element method. Accurate propagation constants and field profiles have been obtained for the modes of the isolated guides and supermodes of the coupled system. The power transfer efficiency between nonidentical coupled optical waveguides is calculated using the coupled mode, the least square boundary residual and the finite element based propagation method.

I. INTRODUCTION

The investigation of coupling between optical waveguides is important for many directional coupler-based devices. Optical directional couplers, made from electro-optic materials, are the basis of several guided-wave devices including switches and modulators. The refractive index of the waveguide material changes due to the applied modulating field, which in turn affects the propagation constants of the two individual guides, the phase matching between them and the coupling length. When the change of refractive indices in the two guides are not identical due to unequal change of refractive indices in two coupled guides, then the power transfer efficiency deteriorates due to lack of phase matching between the guides. All these effects, combined together, change the total power transfer from one guide to another as a function of the applied potential, for switches and modulators. In this paper, the power coupling efficiency between two optical waveguides is presented using the finite element method along with the improved coupled mode approaches[1,2,3].

II. MODAL SOLUTION BY FINITE ELEMENT

The finite element method[4] has already been established as one of the most powerful methods to obtain modal solutions for a wide range of optical waveguides. In the finite element method, the region of concern is subdivided into a patchwork of a number of subregions called elements. These elements each can be of various shapes, such as triangles or rectangles or even having curved sides and they can be of various sizes, to suit the device to be modelled. Using many elements, any cross-section with a complex boundary and various refractive index profiles can be accurately approximated. Each element can also have a different loss or gain factor, different anisotropy or different nonlinearity, so a wide range of practical waveguides can be considered. The vector **H**-field formulation, which has been extensively used for the solution of a variety of optical waveguide problems, can be written as[5]:

$$\omega^2 = \frac{\int (\nabla \times H)^* \cdot \hat{\varepsilon}^{-1} (\nabla \times H) d\Omega + \alpha \int (\nabla \cdot H^*)(\nabla \cdot H) d\Omega}{\int H^* \cdot \hat{\mu} H d\Omega} \tag{1}$$

The second part in the numerator imposes divergence conditions of the field in a least squares sense and eliminates possibility for spurious solutions. The above equation, being a vector formulation, this is "exact-in-the-limit", and particularly suitable for dielectric waveguides as the magnetic field is naturally continuous over the waveguide cross-section. In this formulation, $\hat{\varepsilon}$ can be tensor, so the formulation is applicable to consider electro-optic and acousto-optic effects in optical waveguides and directional couplers. This method can provide accurate propagation constants and modal field profiles for isolated guides and supermodes for coupled structures even when the guides are strongly coupled or nonidentical[3].

III. THE COUPLED MODE APPROACH

Once the modal properties of the isolated modes and supermodes are known, then the power transfer efficiency between the guides can be calculated by using the coupled mode approach. Recently there has been extensive research work to improve the traditional coupled mode approaches, among them the research of Hardy and Streifer,[1] Marcatili[2] and Chuang[3] can be mentioned. In this work, first the accurate propagation constants and transverse modal field profiles are obtained by using the finite element method. Then the mode overlap, C_{ij}, and the coupling coefficients, K_{ij}, are calculated from the modal field profiles using the coupled mode approach. Finally the power transfer efficiency is calculated from the various coupling parameters.

IV. LEAST SQUARES BOUNDARY RESIDUAL METHOD

Since the FEM can provide accurate solutions for the supermodes of the coupled system, an alternative to the coupled mode approach as described in section III, the Least Squares Boundary Residual (LSBR) method has been applied in this paper to directional coupler problems. This procedure is used to find the power carried by the even and odd supermodes for a given incident power in guide "*a*" or "*b*". Here it is assumed a single isolated waveguide section, section **I**, is butt coupled to the directional coupler section, section **II**, as shown in Fig. 1. The main objective is to calculate the amplitudes of the even and odd modes b_1, b_2 respectively in section **II**. This approach is also better than the point matching methods because the error integral is evaluated over the discontinuity interface, rather than just field matching at some specific points.

More detailed discussion on the Least Square Boundary method can be found in our earlier work [6]. Briefly, the LSBR method looks for a stationary solution to satisfy the continuity conditions of both the tangential fields namely, E_t and H_t, in a least square sense by minimizing the error function, J, where

$$J = \int \left| E_t^I - E_t^{II} \right|^2 + \alpha Z_o^2 \left| H_t^I - H_t^{II} \right|^2 d\Omega \qquad (2)$$

where E_t^I, E_t^{II} and H_t^I, H_t^{II} are the transverse electric and magnetic fields in sections **I** and **II** respectively. Z_o is the free-space wave impedance and is a dimensionless weighting factor. It can be shown that the minimum criterion of equation (2) reduces to the following linear equation [6] :

$$\mathrm{Cx} = \mathrm{v} \qquad (3)$$

where [C] is square matrix generated from the eigenvectors and {v} is an array due to the incident mode.

The solution of this equation gives in {x} the required approximate coefficients of a_i and b_i. These constitute one column of the scattering matrix, corresponding to the chosen incident mode.

The eigenvalues and eigenvectors used are first generated by our vector FEM program. The eigenvectors are given by the nodal values of the three components of the vector **H** field for each mode. From these nodal **H** fields, the vector **E** field over each element can be calculated by applying Maxwellûs equation. Many modal eigenvalues and eigenvectors for both sides of the discontinuity plane are used as the input to the LSBR program. The LSBR program calculates the error function J and minimizes the error criterion (2) with respect to each value of a_i and b_i for any given incidence, by solving a homogeneous linear equation (3). Solving this equation will give the unknown column vector {x} consisting of the unknown reflected and transmitted coefficients of all the modes considered in the analysis. The singular value decomposition algorithm has been used to solve the linear equation, (3). To improve the numerical efficiency, the FE nodal points in section **I** are matched with the nodal points of section **II** across the discontinuity interface. In this case there is no need to generate the nodal **E** fields, as the electric field part of the integral J in equation (2) can be calculated directly from the nodal **H** field values.

V. RESULTS

In this example, a titanium-diffused $LiNb0_3$ electro-optic directional coupler switch is considered together with its simplified equivalent planar structure. The unperturbed guides are 2.0 μm wide and with the refractive index 2.2, when no modulation is applied. The separation region between the guides is μm wide with a refractive index in this region and as well as in the two cladding regions of 2.19. The operating wavelength is 1.3 μm. In this work it is assumed that when a positive modulation field is applied, the refractive index in the left guide is increased by $\Delta n/2$ and decreased by an equal amount in the right guide due to the opposite sign of the electric field, and the guides are no longer identical. Although the refractive index change due to the electro-optic effect, Δn_i, can be tensor and proportional to the modulating field components, the variational formulation given by equation (1) can handle this, but in this planar example only an isotropic refractive index change is considered.

The electric field profiles for the first TE supermode are shown in Fig. 2 for $\Delta n = 0$ and 0.002 when the guide separation, s, is 1.9 μm. When no modulation is applied, $\Delta n = 0$, the two guides are identical and the even supermodes are symmetrical. However, when a modulation is applied, it can be observed that the first supermode is the deformed even-like mode with more power confined in the left guide which has a higher refractive index than the right guide. This deformation is more prominent when the guide separation, s, is increased.

Table 1. show the comparison of the finite element solutions with the analytical solutions for the even and odd TE supermodes of the coupled structure. Here β_e and β_o are propagation constants for the even and odd supermodes of the coupled guides. The analytical solutions (AN) are obtained by finding roots of the transcendental equation due to the field matching at the dielectric interfaces. The finite element (FEM) solutions are obtained by using 4000 mesh divisions. It takes about 10 seconds to find modal solutions on a SUN Sparcstation 2 for this mesh refinement. Table 1 shows the excellent agreement of the finite element results with the analytical results and if required, the accuracy can be further improved by using an even finer mesh.

The coupling length decreases with the applied modulation since the propagation constant difference, $\Delta\beta$, between the two isolated waveguides increases. Propagation constants of two supermodes can also be calculated from the unperturbed modes of the two isolated guides using the coupled mode approach. Fig. 3 shows that the coupling length variations with n using the analytical method [AN], the finite element method [FEM] and using the coupled mode approach [CMA]. The analytical results and the finite element results are identical and cannot be distinguished one from another. The results using the coupled mode approach [3] [CMA] shows slight disagreement. It can be observed that the coupling length is 296 μm when $\Delta n = 0.002$ compared to 583 μm at $\Delta n = 0$.

Fig. 4 shows the variation of the coupling coefficients , K_{ij} and C_{ij} by applying the coupled mode approach[3]. It can be noticed at $\Delta n = 0$, $K_{ab} = K_{ba}$, whereas when $|\Delta n|$ increases K_{ab} increases and K_{ba} decreases. It may also be observed that C_{ab} and C_{ba} both increase with $|\Delta n|$ and C_{ab} and C_{ba} are nearly identical.

Fig. 5 shows the variation of the supermode coefficients with Δn for different separations between the two guides, s. It can be seen that the coefficient of the even supermode, b_1, decreases with Δn whereas that of the odd supermode, b_2, increases. It can be also noticed that at large separation, such as $s = 6.0$ μm, b_1 and b_2 are identical at $\Delta n = 0$ but $b_1 \rightarrow 0$ and $b_2 \rightarrow 1.0$ rapidly as n increases.

Fig. 6 shows the evolution of optical wave propagation along the axial direction when $\Delta n = 0$, and in this case the guides are identical. The initial power was launched in guide b, and at a distance equal to the coupling length, L_{co}, most of the power has been transferred to guide a. Fig 7 shows the evolution of optical wave propagation along the axial direction when $\Delta n = 0.0015$ and the guides are not identical. It can be observed that at the coupling length, L_c, which is smaller than L_{co}, only part of the incident power in guide "b".

Fig. 8 shows the variation of the maximum power transferred from guide b to guide a with the change of refractive index difference, Δn, between the guides. It can be seen that

TABLE 1 The comparison of finite element solutions (FEM) with analytical solutions (AN) for β_e and β_o.

Δn	AN		FEM	
	β_e	β_o	β_e	β_o
0.0000	13.01634	13.01096	13.01631	13.01096
0.0004	13.01649	13.01081	13.01649	13.01081
0.0008	13.01691	13.01040	13.01691	13.01040
0.0010	13.01719	13.01013	13.01719	13.01013
0.0015	13.01803	13.00930	13.01803	13.00930
0.0020	13.01900	13.00841	13.01900	13.00841

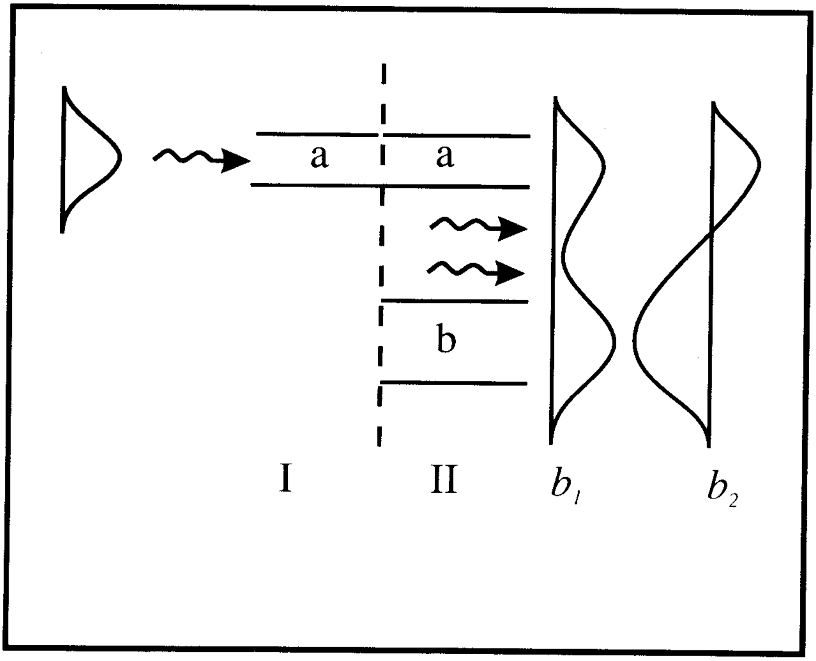

Fig.1. Butt coupling of an isolated guide to the directional coupler section.

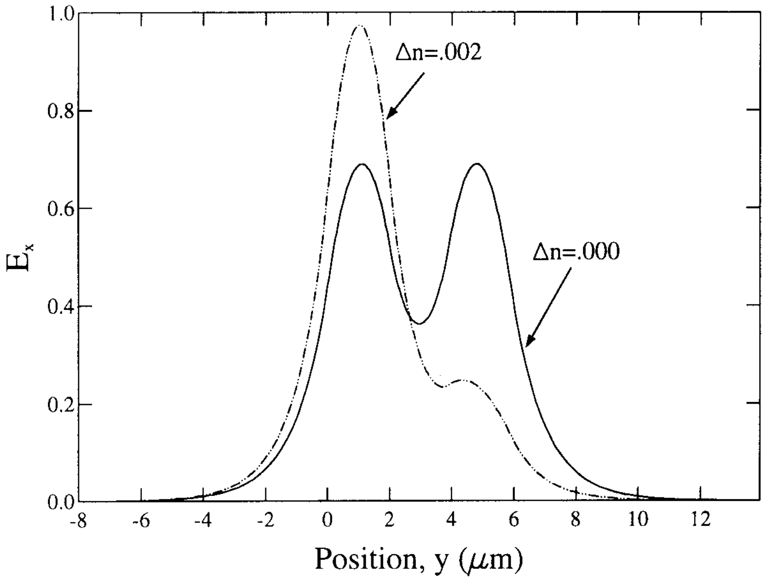

Fig.2. Variation of the field profiles for the first TE supermode, for Δn = 0 and 0.002.

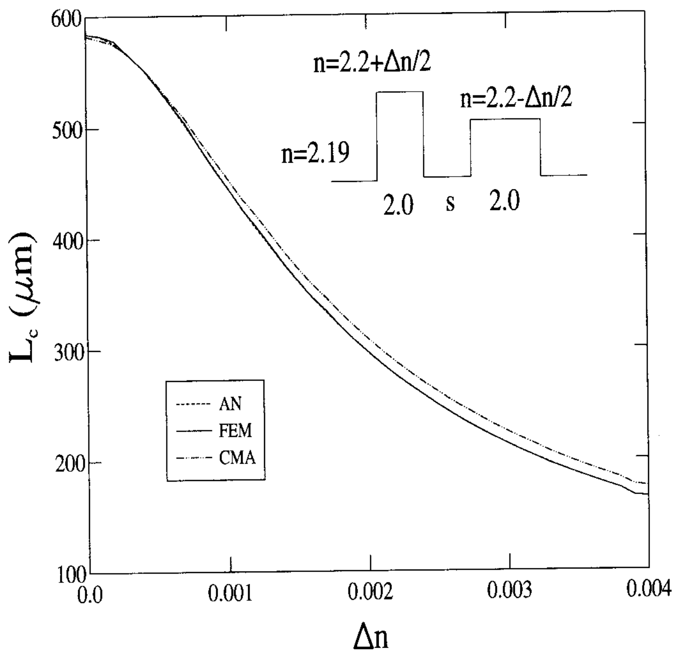

Fig.3. Variation of the coupling length with Δn, using different procedures.

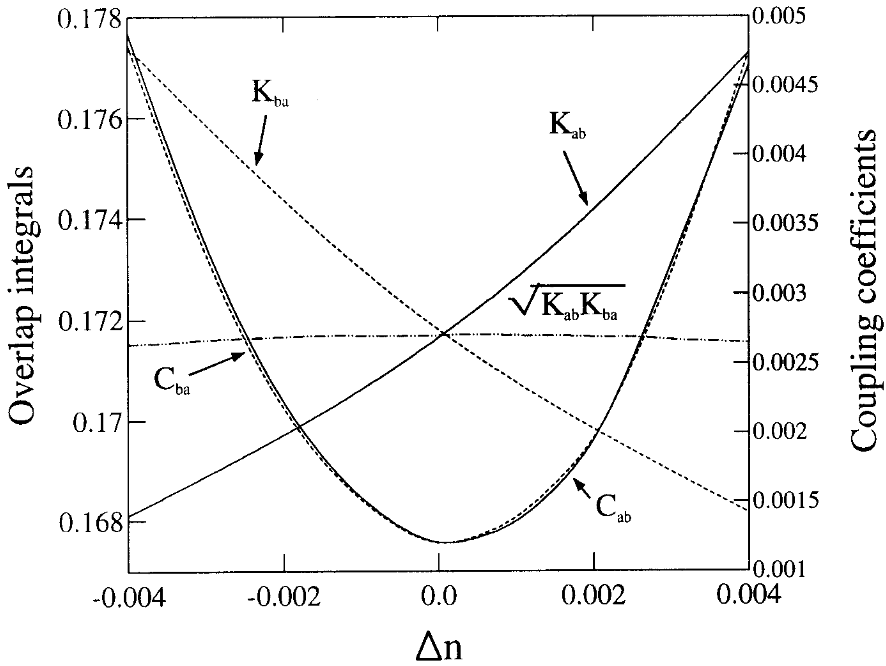

Fig.4. Variation of the overlap integral and coupling coefficients with Δn.

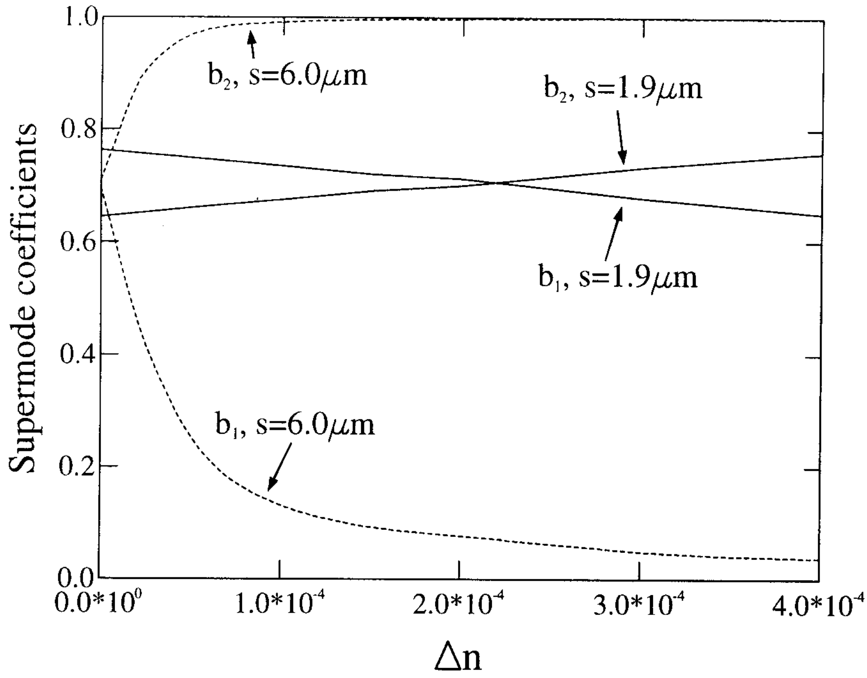

Fig.5. Variation of the supermode coefficients with Δn.

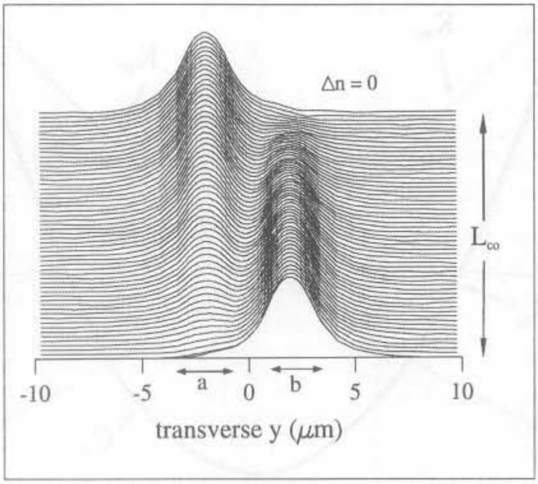

Fig.6. Propagation of optical power along the axial direction when $\Delta n = 0$.

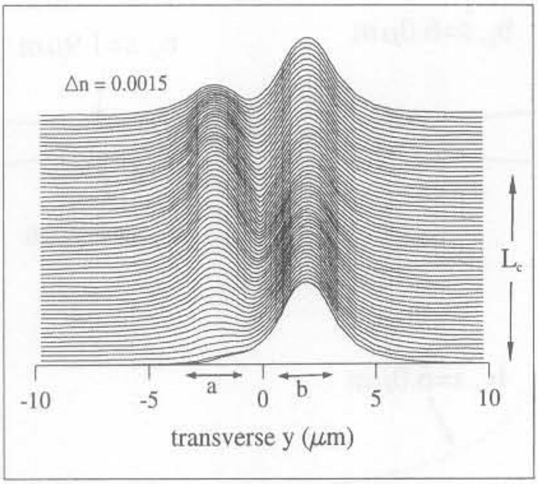

Fig.7. Propagation of optical power along the axial direction when $\Delta n = 0.0015$.

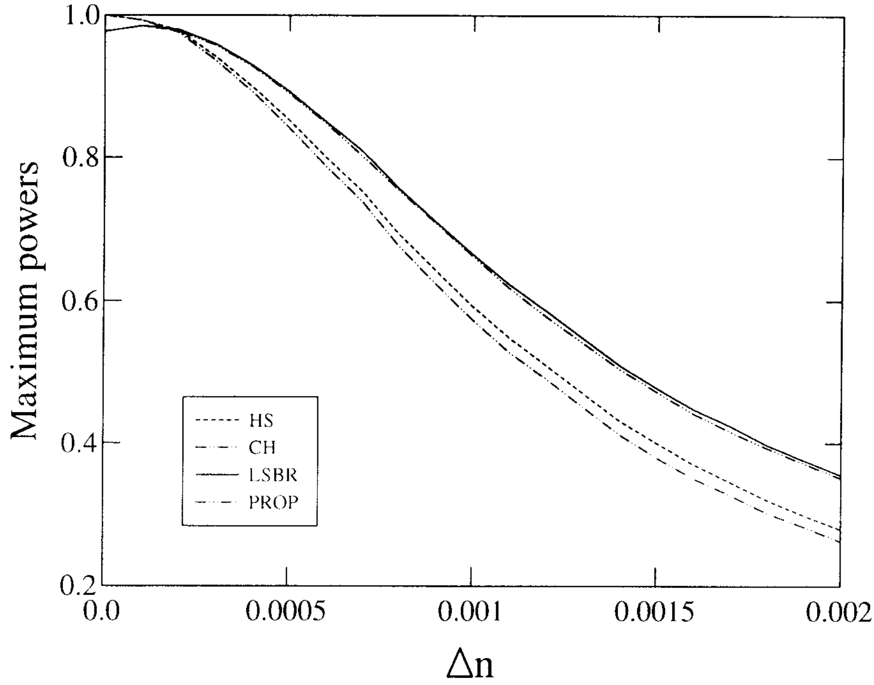

Fig.8. Maximum power transfer between two coupled waveguides.

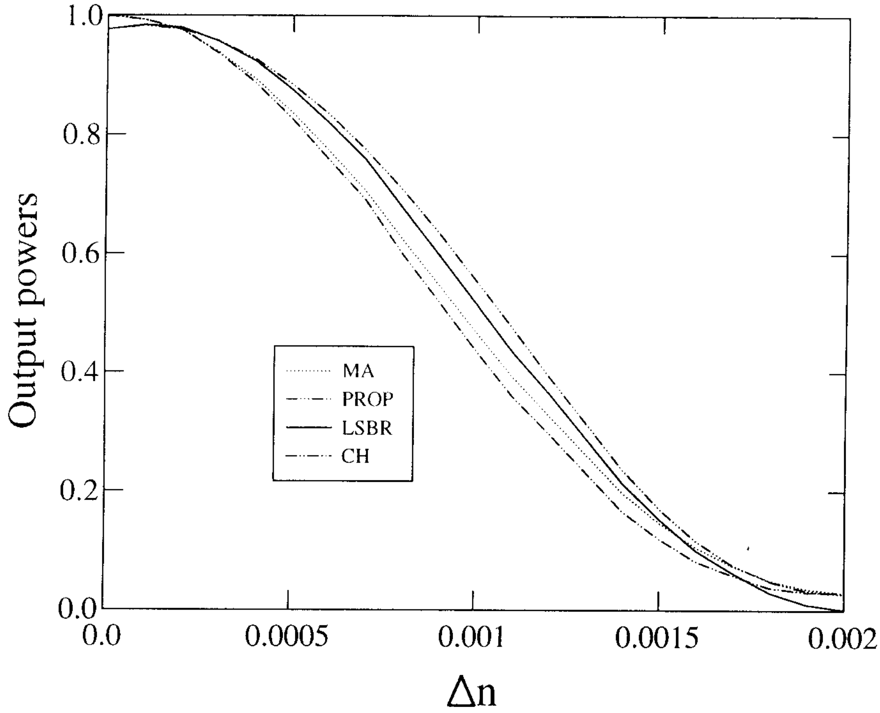

Fig.9. Variation of the coupled output power in guide *a* with the refractive index change Δn.

results obtained using the LSBR and finite element propagation method agree very well but the results from the coupled mode approaches (CH[3] and HS[1]) underestimate the maximum power transfer.

Fig. 9 shows the variation of the output power transfer from guide *b* to guide *a* with the change of refractive index, n, between the guides, when the device length is kept fixed at $L = L_{co} = 583\ \mu m$, which is the coupling length when no modulation is applied. Results obtained using the LSBR approach show that the power efficiency becomes nearly equal to unity, only when the guides are weakly coupled. It can be seen that the results agree reasonably well for all the approaches used. Here, the power transfer efficiency is significantly lower than the maximum power transfer, as shown in Fig. 8. This reduced power transfer is due to the additional effect of the coupling length mismatching as the value of L_c changes with n, whereas the device length is kept fixed.

VI. CONCLUSION

The finite element analysis has been seen to provide accurate results for weakly or strongly coupled identical or nonidentical waveguides. Here the results are restricted to the TE modes in planar waveguides to enable a comparison of our results with other published work but it can be stressed that this numerical procedure is equally valid for hybrid modes in coupled waveguides with two dimensional confinement, anisotropic refractive indices and diffused profiles. The application of coupled mode theory, along with the accurate eigenvectors and eigenvalues obtained by the finite element method, can provide the power transfer ratio between such practical coupled waveguides. Important applications are in the design of directional coupler-based devices, including passive and active filters, modulators and switches incorporating electro-optic, elasto-optic and nonlinear phenomena.

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