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SCALAR PAIR PRODUCTION BY NAMBU STRINGS : ANGULAR CORRELATIONS

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ABSTRACT

Pair production of oppositely charged scalar particles from charged and neutral Nambu strings are investigated. The strings are taken to be the circularly oscillating closed strings, as perhaps the simplest solution of the Nambu action.For the charged string the process proceeds via the couplings of the charged currents through the exchange of the photon and in the case of the neutral string the energy-momentum tensors are coupled via graviton exchange. In particular the angular correlations of the pair-produced scalar particles are calculated and are found to be qualitatively the same for both the emissions from the charged and neutral strings. The angular correlations being positive and increase rapidly with energy indicating that the production is jet-like at high energies. At any particularenergy the angular correlation for the scalar particles produced from the charged string is higher than that for the particles from the neutral string and the difference is quite pronounced at low energies.

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INTRODUCTION

In this paper we investigate the production of oppositely charged scalar particle-antiparticle pair from both the charged and neutral Nambu strings. The string arises from the Nambu action [2,3] as a solution of a circularly oscillating closed string [1,4] as perhaps the simplest object generalizing emissions from point-like particles within the framework of quantum field theory. We consider the case of scalars with only electromagnetic and gravitational interactions. We give a detailed analytical and numerical analysis of pair production of oppositely charged scalar particles with the couplings due to the charged current *via* the exchange of photon and arising from their energy-momentum tensors via the exchange of a graviton in the cases of charged and neutral Nambu strings respectively. Given that the process of pair production has occurred, we compute the conditional probability density in question. Finally the expressions of the angular correlations of the pair are calculated. Due to the relatively large angular correlations obtained from both the charged and neutral strings, it is argued that higher order effects to either correlations might not be significant.

The statistical term for the study of angular correlations is the one defined [5] as the average of the cosine of the angle between the directions of momenta of two outgoing particles in a given process. A positive correlation indicates that the two particles tend to travel, in a statistical sense, in the same directions, while a negative one indicates that they tend to travel in opposing directions. For a study of some fundamental processes in field theory of angular correlations see, for example, Refs. [1,5-7]. Sect. 2 considers the dynamics of the string giving the expressions for the electromagnetic current of the charged string and the energy-momentum tensor of the neutral string. The expressions for the conditional probability density of the processes in question are given in Sect.3. Finally, the angular correlations are studied in Sect.4. Contributions of higher order effects are also dicussed.

Dynamics of the String

The dynamics of the string is described as follows. The trajectory of the string is described by a vector function $\mathbf{R}(\sigma, t)$, where σ parametrizes the string. The equation of motion of the closed string is taken to be [2-4]

$$\frac{\partial^2}{\partial t^2}\mathbf{R} - \frac{\partial^2}{\partial \sigma^2}\mathbf{R} = 0, \tag{1}$$

with constraints

$$\partial_t\mathbf{R} \cdot \partial_\sigma\mathbf{R} = 0, (\partial_t\mathbf{R})^2 + (\partial_\sigma\mathbf{R})^2 = 1, \mathbf{R}(\sigma + \frac{2\pi}{m}, t) = \mathbf{R}(\sigma, t) \tag{2}$$

where m is taken to be the mass of of the scalar particle, and $\partial_t \mathbf{R} = \frac{\partial \mathbf{R}}{\partial t}$, $\partial_\sigma \mathbf{R} = \frac{\partial \mathbf{R}}{\partial \sigma}$. The general solution to Eqs:(1),(2) is

$$\mathbf{R}(\sigma, t) = \frac{1}{2}[\mathbf{A}(\sigma - t) + \mathbf{B}(\sigma + t)], \tag{3}$$

where $\mathbf{A}$, $\mathbf{B}$ satisfy, in particular, the normalization conditions $(\partial_\sigma \mathbf{A})^2 = (\partial_\sigma \mathbf{B})^2 = 1$. For the system Eqs:(1)-(3) we consider a solution of the form [1,4] :

$$\mathbf{R}(\sigma, t) = \frac{1}{m}(\cos m\sigma, \sin m\sigma, 0) \sin mt, \tag{4}$$

describing a radially oscillating circular string.

The general expression for the electromagnetic current, J^μ, associated with a dynamical *charged* string is defined by (cf.[1])

$$J^\mu = \frac{Qm}{2\pi} \int_0^{\frac{2\pi}{m}} d\sigma \partial_t R^\mu \delta^3(\mathbf{r} - \mathbf{R}(\sigma, t)), \tag{5}$$

with $\mathbf{r}$ lying in the plane of the string :

$$\mathbf{r} = r(\cos\phi', \sin\phi', 0) \tag{6}$$

where $R^0 = t$, and Q denotes the total charge of the string. On the other hand the energy-momentum tensor of the string is given by [3]

$$T^{\mu\nu} = \frac{m^2}{2\pi} \int_0^{\frac{2\pi}{m}} d\sigma (\partial_t R^\mu \partial_t R^\nu - \partial_\sigma R^\mu \partial_\sigma R^\nu) \delta^3(\mathbf{r} - \mathbf{R}(\sigma, t)) \tag{7}$$

From Eqs:(4),(5), and (7) and using, in the process, the time periodicity condition : $t \to t + \frac{2\pi}{m}$, we obtain after a lengthy process the following expressions for J^μ and $T^{\mu\nu}$:

$$J^\mu(t, \mathbf{r}, z) = \int \frac{dp^0}{(2\pi)} \int \frac{d^2p}{(2\pi)^2} \int \frac{dq}{(2\pi)} e^{i\mathbf{p}\cdot\mathbf{r}} e^{iq\cdot z} e^{-ip^0 t} J^\mu(p^0, \mathbf{p}, q) \tag{8}$$

$$J^\mu(p^0, \mathbf{p}, q) \equiv J^\mu(p^0, \mathbf{p}) = 2\pi \sum_{N=-\infty}^{\infty} \delta(p^0 - mN) B^\mu(\mathbf{p}, N) \tag{9}$$

with

$$\mathbf{p} = p(\cos\phi, \sin\phi, 0) \tag{10}$$

$$B^0(\mathbf{p}, N) = \alpha_n J_n^2(x) \tag{11}$$

$$\mathbf{B}(\mathbf{p}, N) = \frac{mN}{\mathbf{p}^2} \mathbf{p} B^0(\mathbf{p}, N) \tag{12}$$

where

$$\alpha_n \equiv Q(-1)^n \cos(n\pi) \tag{13}$$

$$n = \frac{N}{2} \tag{14}$$

$$x = \frac{|\mathbf{p}|}{2m} \tag{15}$$

$$p^\mu = (p^0, \mathbf{p}, 0) \tag{16}$$

$$T^{\mu\nu}(t, \mathbf{r}, z) = \int \frac{dp^0}{(2\pi)} \int \frac{d^2p}{(2\pi)^2} \int \frac{dq}{(2\pi)} e^{i\mathbf{p}\cdot\mathbf{r}} e^{iq\cdot z} e^{-ip^0 t} T^{\mu\nu}(p^0, \mathbf{p}, q) \tag{17}$$

$$T^{\mu\nu}(p^0, \mathbf{p}, q) \equiv T^{\mu\nu}(p^0, \mathbf{p}) = 2\pi \sum_{N=-\infty}^{\infty} \delta(p^0 - mN) B^{\mu\nu}(\mathbf{p}, N) \tag{18}$$

$$B^{00} = \beta_n J_n^2(x) \tag{19}$$

$$B^{0a} = \beta_n \frac{p^0 p^a}{\mathbf{p}^2} J_n^2(x), a = 1, 2 \tag{20}$$

$$B^{ab} = \beta_n A_n \delta^{ab} + \beta_n E_n \frac{p^a p^b}{\mathbf{p}^2}, a, b = 1, 2 \tag{21}$$

$$B^{\mu 3} = 0 \tag{22}$$

where

$$A_n = \frac{1}{4}[J_{n+1}^2(x) + J_{n-1}^2(x) - 2J_{n-1}(x)J_{n+1}(x)] \tag{23}$$

$$E_n = J_{n+1}(x) J_{n-1}(x) \tag{24}$$

$$\beta_n = m(-1)^n \cos(n\pi) \tag{25}$$

The $J_n(x)$ from Eq.(11) onward denotes the Bessel function of order n with argument x defined in Eq.(15).

We explicitly verify the conservation law $\partial_\mu J^\mu = 0$, $\partial_\mu T^{\mu\nu} = 0$ by checking the correctness of the expressions

$$p_\mu B^\mu = 0, p_\mu B^{\mu\nu} = 0 \tag{26}$$

where

$$p^0 = 2mn = mN = -p_0 \tag{27}$$

is the total energy of the monoenergetic pair of the oppositely charged scalar particles each of mass m.

The Conditional Probability for the Processes

First, we consider the case of the charged Nambu string. To lowest order in the fine structure constant the amplitude for the production of a pair of oppositely charged scalar particles of momenta **P**, **P′** by the string is given by (up to an unimportant overall multiplicative factor for the problem at hand) :

$$\frac{J^{\mu}(p)(P_{\mu}+P'_{\mu})}{(P+P')^2}, \tag{28}$$

where $J^{\mu}(p)$ is defined in Eqs:(9)-(16). We are interested in the relative probability density of the emission of the pair per unit time [1] averaged over a period of length $\frac{2\pi}{m}$. For a monoenergetic pair $P^0 = P'^0$. By defining, in the process,

$$\mathbf{P} = m\sqrt[2]{n^2-1}(\cos\phi_1 \sin\theta_1, \sin\phi_1 \sin\theta_1, \cos\theta_1) \tag{29}$$

$$\mathbf{P}' = m\sqrt[2]{n^2-1}(\cos\phi_2 \sin\theta_2, \sin\phi_2 \sin\theta_2, \cos\theta_2) \tag{30}$$

$$\phi = \phi_1 - \phi_2 \tag{31}$$

$$P^0 = mn = P'^0, p^0 = 2mn \equiv mN \tag{32}$$

we obtain for the *unnormalized* conditional probability density that the pair emerges, each with the energy nm and momenta in directions specified by the unit vectors : $(\cos\phi_i \sin\theta_i, \sin\phi_i \sin\theta_i, \cos\theta_i), i = 1,2$, given that the process has occurred, the expression :

$$f_n(\theta_1,\theta_2,\phi) = \frac{(\sin^2\theta_1 - \sin^2\theta_2)^2 J_n^4(x)}{a_n^2 b_n}, \tag{33}$$

where $n = 2,3,\ldots$ and

$$a_n = [\sin^2\theta_1 + \sin^2\theta_2 + 2\cos\phi\sin\theta_1\sin\theta_2)], \tag{34}$$

$$b_n = [2-(n^2-1)(\cos\phi\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2 - 1)]^2, \tag{35}$$

and x defined in Eq.(15) may be rewritten in terms of the angles as :

$$x = \frac{1}{2}(n^2-1)^{1/2}[\sin^2\theta_1 + \sin^2\theta_2 + 2\cos\phi\sin\theta_1\sin\theta_2]^{1/2} \tag{36}$$

Next we come to the pair production from neutral Nambu string. To lowest order in the gravitational coupling constant the amplitude for the production of a pair of oppositely charged scalar particles of momenta **P**,**P′** by a neutral string is given by (up to an unimportant multiplicative factor) [9] :

$$T^{\sigma\lambda}\frac{[g_{\sigma\mu}g_{\lambda\nu} - \frac{1}{2}g_{\sigma\lambda}g_{\mu\nu}]}{(P+P')^2}T^{\mu\nu}_{pair} \tag{37}$$

where $T^{\mu\nu}_{pair}$ is the energy-momentum tensor for the produced scalar particle pair and is :

$$T^{\mu\nu}_{pair} \propto \frac{-1}{2}(P^\mu P'^\nu + P'^\mu P^\nu) + \frac{g_{\mu\nu}}{2}(PP' - m^2), \tag{38}$$

$$P + P' = (2mn, \mathbf{p}, q), \tag{39}$$

and $T^{\sigma\lambda}$ is the energy-momentum associated with the neutral string defined through Eqs: (18)-(25). A lengthy analysis then shows from consideration of the amplitude in Eq.(37) for the emission of the pair per unit time averaged over a period of length $\frac{2\pi}{m}$, that the conditional probability density in question for the pair to emerge with momenta in the directions $(\cos\phi_i \sin\theta_i, \sin\phi_i \sin\theta_i, \cos\theta_i), i = 1, 2$, given that the process has occurred for the emission of a monoenergetic pair each with energy mn, is given by

$$F_n(\theta_1, \theta_2, \phi) = |H_n(\theta_1, \theta_2, \phi)|^2, \tag{40}$$

where $n = 2, 3, ...$ and

$$H_n(\theta_1, \theta_2, \phi) = \frac{G_n - 4(n^2-1)[A_n\Omega + (E_n/a_n)(\Omega + \sin^2\theta_1)(\Omega + \sin^2\theta_2)]}{2 - (n^2-1)(\Omega + 2\cos\theta_1\cos\theta_2 - 1)} \tag{41}$$

where A_n, E_n, and G_n are sums and products of Bessel functions of integral order, given respectively by, Eq.(23), Eq.(24), and

$$G_n = [J^2_{n+1}(x) + J^2_{n-1}(x) + 2(2n^2-1)^2 J^2_n(x)], \tag{42}$$

with argument x which is defined in Eq.(36) and finally a_n and Ω are functions of θ_1, θ_2, ϕ given respectively by Eq.(34) and

$$\Omega = \cos\phi \sin\theta_1 \sin\theta_2 \tag{43}$$

Angular Correlations

The angular correlations for the monoenergetic oppositely-charged scalar particle pair pair-produced by the Nambu string,$\langle c_n \rangle$, at the energy n is given by

$$\langle c_n \rangle = \frac{\int_0^\pi \sin\theta_1 d\theta_1 \int_0^\pi \sin\theta_2 d\theta_2 \int_0^{2\pi} d\phi \mathcal{F}_n(\theta_1, \theta_2, \phi)[\cos\phi\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2]}{\int_0^\pi \sin\theta_1 d\theta_1 \int_0^\pi \sin\theta_2 d\theta_2 \int_0^{2\pi} d\phi \mathcal{F}_n(\theta_1, \theta_2, \phi)} \tag{44}$$

where $\mathcal{F}_n(\theta_1, \theta_2, \phi)$ is the conditional probability density for the process under investigation and

$$[\cos\phi\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2] = \frac{\mathbf{P}\cdot\mathbf{P}'}{|\mathbf{P}||\mathbf{P}'|} \tag{45}$$

gives the cosine of the angle between the momenta of the two out going particles. We note that the expression on the left-hand side of Eq.(45) can take on positive as well as negative values.

We denote the angular correlations for production from charged and neutral strings respectively by $\langle c_n\rangle_{charged}$ and $\langle c_n\rangle_{neutral}$. Then $\langle c_n\rangle_{charged}$ and $\langle c_n\rangle_{neutral}$ are obtained from Eq.(44) using respectively $f_n(\theta_1, \theta 2, \phi)$ from Eq.(33) and $F_n(\theta_1, \theta_2, \phi)$ from Eq.(40) for $\mathcal{F}_n(\theta_1, \theta_2, \phi)$.

The angular correlations , $\langle c_n\rangle_{charged}$ and $\langle c_n\rangle_{neutral}$, for various energies starting from $n = 2$ are numerically evaluated and the results are shown versus n in the figure below. It is to be noticed that, for both the pair-productions from charged and neutral strings, $\langle c_n\rangle$'s are strictly positive even though a priori $\langle c_n\rangle$ as given by Eq.(44) can take on either positive or negative values. Qualitatively, $\langle c_n\rangle_{charged}$ and $\langle c_n\rangle_{neutral}$ show the same behaviour : they are both positive and after an initial steep increase with energy approach asymptotically 1. The strict positivity of $\langle c_n\rangle$ for both cases indicates clearly that the produced scalar particle-antiparticle pair tend, in a statistical sense, to travel in the same directions. At high energies, i.e. large values of n, $\langle c_n\rangle_{charged}$ and $\langle c_n\rangle_{neutral}$ assume large values (close to one) and the emission of the pair become jet-like.

A comparison of the two processes shows that $\langle c_n\rangle_{neutral}$ is consistently smaller than $\langle c_n\rangle_{charged}$ at all energies. At the threshold energy, $n = 2$, the angular correlations attain the lowest values and give rise to the the following numerical lower bounds : $\langle c_n\rangle_{neutral} \geq 0.368$, $\langle c_n\rangle_{charged} \geq 0.473$. As the energy increases the angular correlations increase with $\langle c_n\rangle_{neutral}$ reaching a large value of 0.940 and $\langle c_n\rangle_{charged}$ reaching a value of 0.987 for $n = 80$ corresponding a total energy of the pair : $p^0 = 2mn = 160m$. These results indicate the possibility of establishing whether the emitting string is neutral or charged from the study of the angular correlation of the produced particle-antiparticle pair. The previous studies of the monoenergetic e^+e^- pair productions from *charged* and *neutral* Nambu strings [1] gave qualitatively similar results, but with greater difference between the correlations at a given energy. In particular, the pair productions are also *jet-like* at high energies. This together with our present results show that pair productions of monoenergetic oppositely charged particle-antiparticle pairs from both *charged* and *neutral* Nambu strings tend to be *jet-like* at high energies for the cases of scalar and Dirac particles.

The angular correlations $\langle c_n\rangle$ for both types of strings up to leading electromagnetic and gravitational corrections are of the form : $c_n 1 + O(\alpha)$, where $\alpha \cong 1/137$ is the electromagnetic fine-structure constant and $c_n 1$ are the values of the correlations computed above. The relatively large values for $\langle c_n\rangle$ obtained, therefore allows one to have some confidence in the correctness of our conclusions from lowest order contributions alone and it is safe to assume that higher order corrections should not change significantly, at least for energies not too small, the numerical values computed in our investigations and, in particular, would not change the positivity

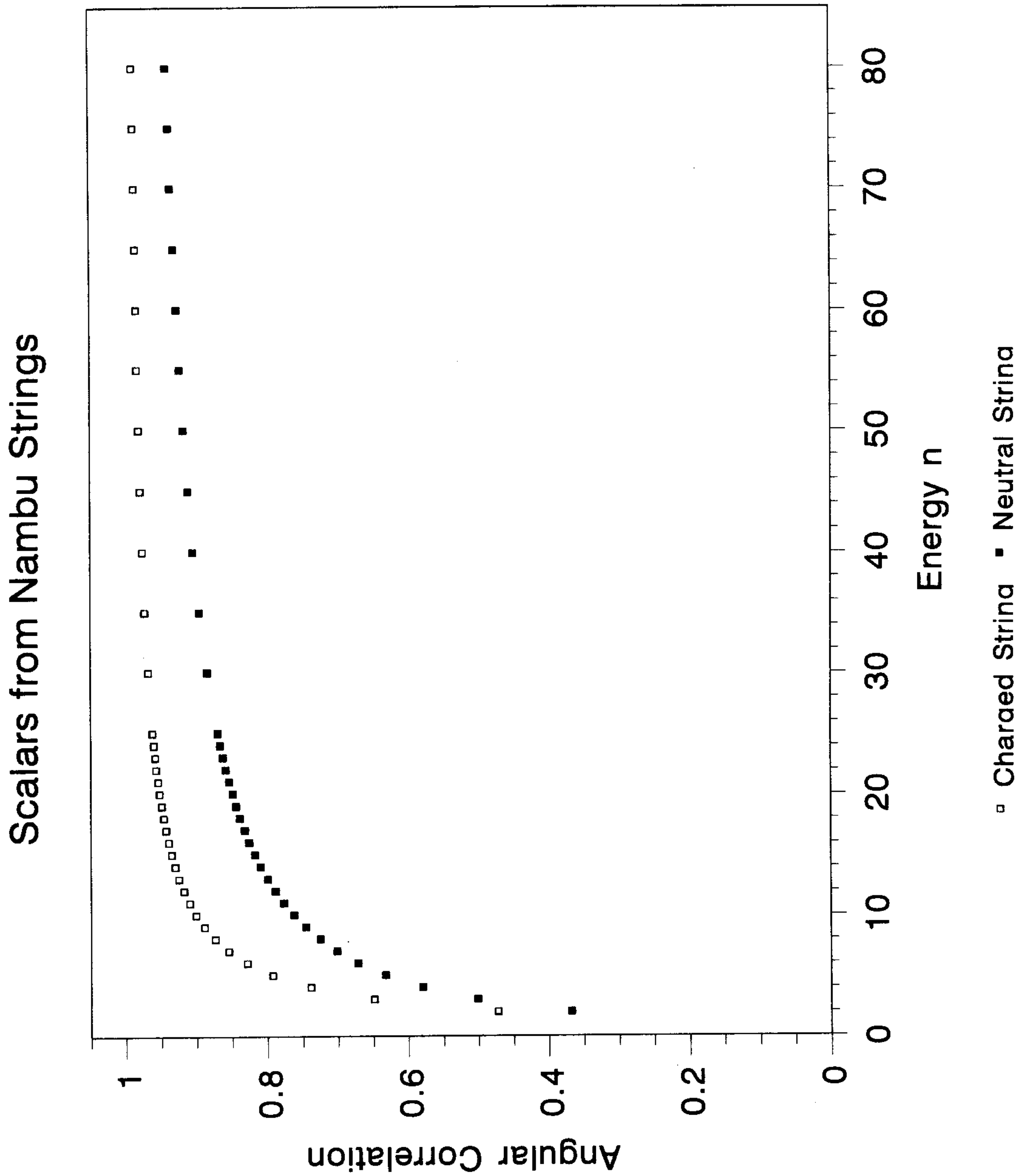

Fig.1 Plots of the angular correlations $(C_n)_{charged}$ (□) and $(C_n)_{neutral}$ (■) vs. *n* the energy of each of the particle in units of the mass of the particle.

property of the correlations. The treatment of any recoil of the string is beyond the scope of the present work.

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