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Numerical solutions of MHD rotating flow and heat transfer over a permeable shrinking sheet

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ABSTRACT: This study deals with the steady magnetohydrodynamics rotating boundary layer flow and heat transfer of a viscous fluid over a permeable shrinking sheet. Similarity transformations have been used for reducing the partial differential equations into a system of ordinary differential equations. The transformed ordinary differential equations are solved numerically using a finite-difference scheme. The effects of the magnetic parameter M, suction parameter s, rotating parameter λ, and Prandtl number Pr on the velocity and temperature fields are presented graphically and discussed in detail.

KEYWORDS: boundary layer, magnetohydrodynamics, viscous fluid, similarity solutions, finite-difference scheme

INTRODUCTION

The study of flow or heat and mass transfer problems due to stretching boundary/surface has many applications in technological processes, particularly in polymer systems involving drawing of fibres and films or thin sheets, production of paper, roofing shingles, insulting material, and others. Since the pioneering work of Sakiadis [1], various aspects of boundary layer flow due to a stretching sheet have been investigated. Crane [2] extended the idea to the two-dimensional problem where the velocity is proportional to the distance from the plate. The uniqueness of the exact analytical solution presented in Ref. 1 is discussed by McLeod and Rajagopal [3]. The study of heat and mass transfer over a stretching sheet subject to suction or blowing was investigated by Gupta and Gupta [4] and Magyari and Keller [5].

In recent years, problems involving magnetohydrodynamics have become increasingly important in industry. Pavlov [6] considered the steady laminar flow of an electrically conducting fluid caused by the stretching of an elastic sheet in the presence of a uniform magnetic field and obtained an exact similarity solution. Anderson [7] obtained an analytically exact solution for MHD flow of a viscoelastic fluid past a stretching surface and he found that the elasticity and magnetic field reduce the boundary layer thickness and increase the skin friction. On the other hand, Nazar et al [8] investigated the unsteady boundary layer flow due to a stretching surface in a rotating fluid while Ishak et al [9] studied the MHD boundary layer flow and heat transfer adjacent to stretching vertical sheet with power law velocity. Wang [10] considered the steady rotating fluid and heat transfer on a stretching sheet while Abbas et al [11] investigated the unsteady MHD flow and heat transfer on a stretching sheet in a rotating fluid.

All the above mentioned investigations deal with the stretching flow problems, but studies on the flow problems due to a shrinking sheet are relatively scarce. Wang [12] was the first to study the unsteady viscous flow induced by a shrinking liquid film. Moreover, Miklavcic and Wang [13] proved the existence and non-uniqueness for steady viscous hydrodynamic flow due to a shrinking sheet for a specific value of the suction parameter. Sajid et al [14] considered the MHD rotating flow of a viscous fluid over a shrinking sheet, while Hayat et al [15] investigated the analytic solution for MHD rotating flow of a second grade fluid past a porous shrinking surface. Later, Faraz and Khan [16] investigated the steady two-dimensional MHD rotating flow of a second grade fluid due to a porous shrinking surface.

The present paper extends the idea in Ref. 14 by including the heat transfer characteristics, and solving it numerically using the Keller-box method. This method was introduced by Keller and Cebeci [17] and has been used by many researchers to solve various boundary layer problems [18–20].

MATHEMATICAL FORMULATION

Consider the steady laminar MHD boundary layer flow of a viscous fluid caused by a 2-d shrinking

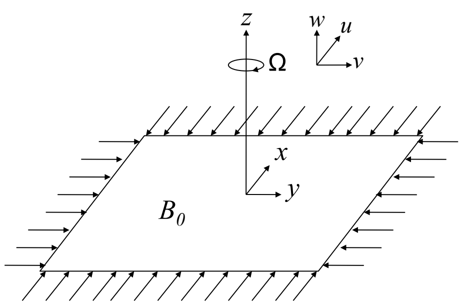

Fig. 1 Physical model and coordinate system.

surface in a rotating fluid. Let u, v, w be the velocity components along the x, y, z directions, respectively, and let Ω be the angular velocity of the rotating fluid in the z-direction (Fig. 1). In addition, a constant magnetic field B_0 is applied in the z-direction. Under the assumption of zero electric field and small magnetic Reynolds number, the boundary layer equations which govern the MHD flow in the absence of pressure gradient (Ref. 14) are

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}-2\Omega v=\nu\frac{\partial^2 u}{\partial z^2}-\frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial v}{\partial x}+w\frac{\partial v}{\partial z}+2\Omega u=\nu\frac{\partial^2 v}{\partial z^2}-\frac{\sigma B_0^2}{\rho}v, \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\frac{\partial^2 T}{\partial z^2}. \tag{4}$$

where $\nu=\mu/\rho$ is the kinematic viscosity, σ the electrical conductivity, ρ the density, α the thermal diffusivity, and T the fluid temperature. The boundary conditions for the equations (1)–(4) are

$$\begin{aligned} u=-ax,\ v=0,\ w=-W,\ T=T_w \ \text{ at } z=0,\\ u\to 0,\ \ v\to 0,\ \ T\to T_\infty \quad \text{as } z\to\infty, \end{aligned} \tag{5}$$

where $a>0$ is the shrinking constant and $W>0$ is the suction velocity. The following similarity transformations were introduced by Sajid et al[14]:

$$\begin{aligned} u=-axf'(\eta),\ v=axg(\eta),\ w=-\sqrt{a\nu}f(\eta),\\ \eta=\sqrt{\frac{a}{\nu}}z,\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \end{aligned} \tag{6}$$

where η is the independent dimensionless similarity variable, and primes denote the differentiation with respect to η, $f(\eta)$ and $g(\eta)$ are the velocity and lateral velocity profiles, respectively, while $\theta(\eta)$ is the temperature profile. Under these similarity transformations, (1) is identically satisfied, while (2)–(4) are reduced to the following:

$$f'''-f'^2+ff''+2\lambda g-M^2f'=0, \tag{7}$$

$$g''-f'g+fg'-2\lambda f'-M^2g=0, \tag{8}$$

$$\frac{1}{\mathrm{Pr}}\theta''+f\theta'=0, \tag{9}$$

The boundary conditions (5) become

$$\begin{aligned} f(0)=s, f'(0)=-1, g(0)=0, \theta(0)=1,\\ f'(\infty)\to 0, g(\infty)\to 0, \theta(\infty)\to 0, \end{aligned} \tag{10}$$

where $s=W/m\sqrt{a\nu}$ is the suction parameter, $M^2=\sigma B_0^2/\rho a$ is the magnetic parameter, $\mathrm{Pr}=\nu/\alpha$ is the Prandtl number and $\lambda=\Omega/a$ is the dimensionless parameter that shows the relationship between rotation rate and the rate of shrinkage.

The local skin friction coefficients in the x- and y-directions and the local Nusselt number are given by

$$\begin{aligned} C_{fx}=\frac{\tau_{wx}}{\rho u^2}, C_{fy}=\frac{\tau_{wy}}{\rho u^2},\\ Nu_x=\frac{xq_w}{k(T_w-T_\infty)}, \end{aligned} \tag{11}$$

where the shear stresses τ_{wx}, τ_{wy} and the heat flux q_w are defined by

$$\begin{aligned} \tau_{wx}=\mu\left(\frac{\partial u}{\partial z}\right)_{z=0}, \tau_{wy}=\mu\left(\frac{\partial v}{\partial z}\right)_{z=0},\\ q_w=-k\left(\frac{\partial T}{\partial z}\right)_{z=0}, \end{aligned} \tag{12}$$

with ν and k being the dynamic viscosity and thermal conductivity, respectively.

Using (6) and (12), (11) becomes

$$\begin{aligned} C_{fx}Re_x^{1/2}=f''(0), \quad C_{fy}Re_y^{1/2}=g'(0),\\ Nu_xRe_x^{-1/2}=-\theta'(0). \end{aligned} \tag{13}$$

RESULTS AND DISCUSSION

The equations (7)–(9) subject to the boundary conditions (10) are solved numerically using an implicit finite difference scheme known as the Keller-box method described in Ref. 17. In order to verify the accuracy of the present method, the numerical results for $f''(0)$, $g'(0)$ and $-\theta'(0)$ are compared with the results obtained in Ref. 10 and Ref. 11 by setting $s=0$, $M=0$, $\mathrm{Pr}=7$ and $f'(0)=1$ (stretching sheet) in the boundary conditions (10). The comparisons, presented in Table 1, are found to be in

Table 1 Comparison of the present values of $f''(0)$, $g'(0)$ and -$\theta'(0)$ with those of Wang [10] and Abbas et al [11].

	Wang [10]			Abbas et al [11]			Present results		
λ	$f''(0)$	$g'(0)$	$-\theta'(0)$	$f''(0)$	$g'(0)$	$-\theta'(0)$	$f''(0)$	$g'(0)$	$-\theta'(0)$
0	-1.0000	0.0000	1.894	-1.0000	0.0000	1.894	-1.0005	0.0000	1.895
0.5	-1.1384	-0.5128	1.850	-1.1384	-0.5128	1.850	-1.1385	-0.5127	1.851
1	-1.3250	-0.8371	1.788	-1.3250	-0.8371	1.788	-1.3250	-0.8371	1.788
2	-1.6523	-1.2873	1.664	-1.6523	-1.2873	1.664	-1.6524	-1.2873	1.669

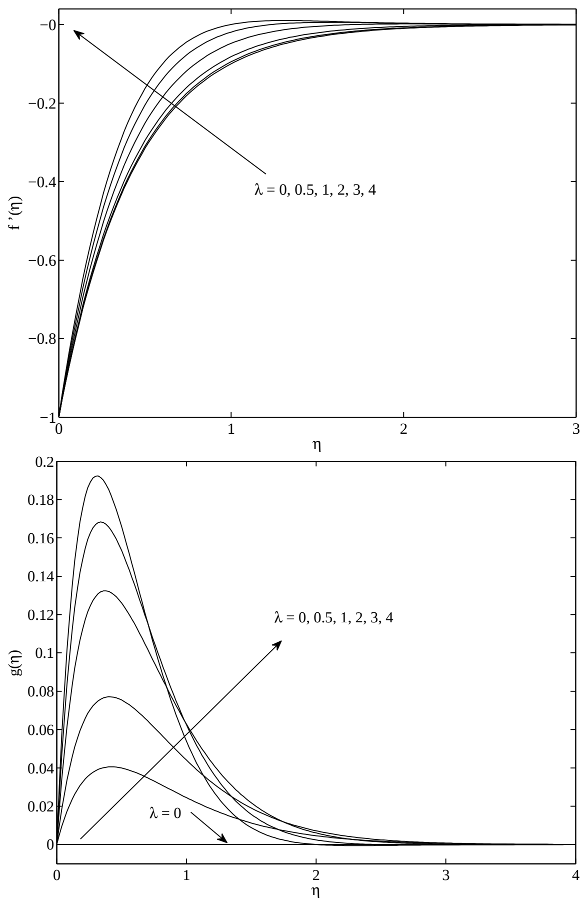

Fig. 2 Effects of the rotating parameter λ on the velocity profiles $f'(\eta)$ and $g(\eta)$ when $s = 3$, $M = 2$, $\mathrm{Pr} = 0.7$.

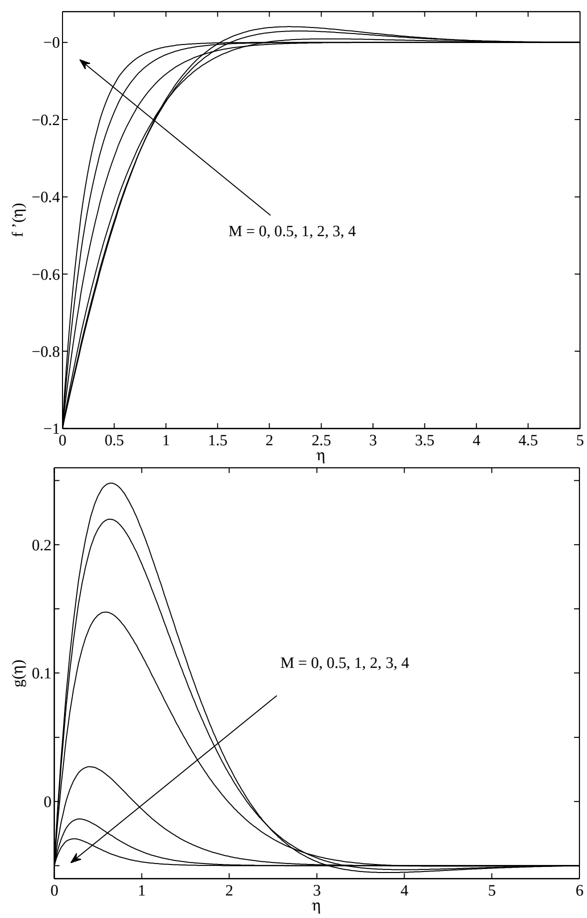

Fig. 3 Effects of the magnetic parameter M on the velocity profiles $f'(\eta)$ and $g(\eta)$ when $s = 3$, $\lambda = 2$, $\mathrm{Pr} = 0.7$.

good agreement, and thus we are confident that the present method is accurate.

Fig. 2 displays the velocity profiles $f'(\eta)$ and $g(\eta)$ for various values of the rotating parameter λ when $s = 3$, $M = 2$, $\mathrm{Pr} = 0.7$. Both profiles show reduction in boundary layer thickness with the increase of the rotating parameter λ. It is found that the velocity increases exponentially for small values of λ and the oscillatory behaviour only occurs for large values of λ. The effects of the magnetic parameter M when $s = 3$, $\lambda = 2$, $\mathrm{Pr} = 0.7$ on the velocity profiles $f'(\eta)$ and $g(\eta)$ are given in Fig. 3 which shows that the boundary layer thickness is smaller as M increases. Meanwhile, graphs of the velocity profiles $f'(\eta)$ and $g(\eta)$ for various values of suction parameter s are given in Fig. 4, which also displays the reduction in boundary layer thickness with the increase of suction parameter s.

Fig. 5 illustrates the influence of the rotating

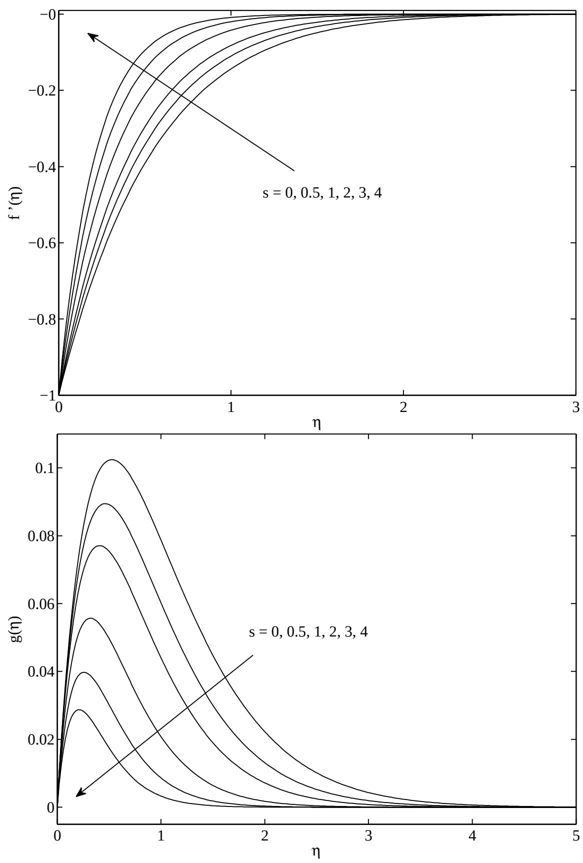

Fig. 4 Effects of the suction parameter s on the velocity profiles $f'(\eta)$ and $g(\eta)$ when $M = 2$, $\lambda = 2$, $\mathrm{Pr} = 0.7$.

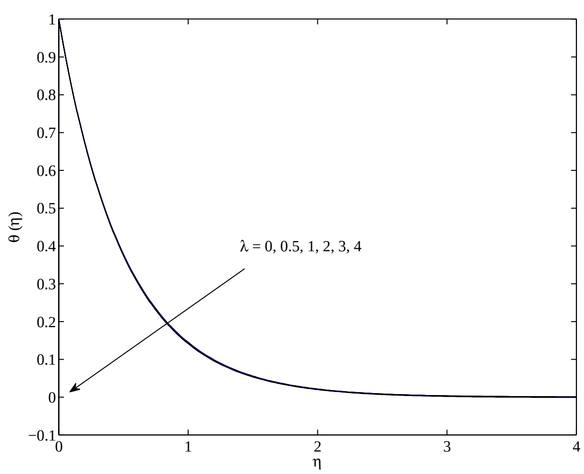

Fig. 5 Effects of the rotating parameter λ on the temperature profiles $\theta(\eta)$ when $M = 2$, $s = 3$, $\mathrm{Pr} = 0.7$.

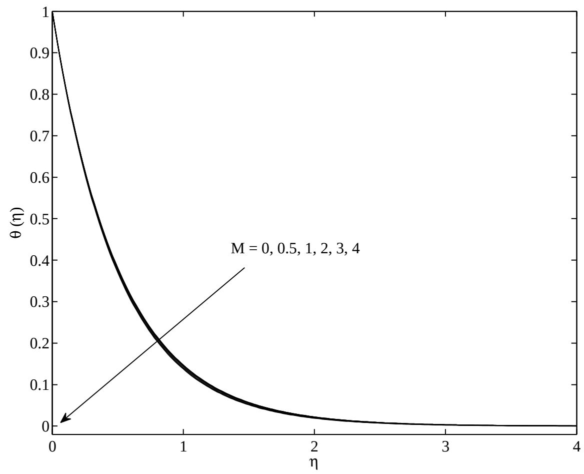

Fig. 6 Effects of the magnetic parameter M on the temperature profiles $\theta(\eta)$ when $\lambda = 2$, $s = 3$, $\mathrm{Pr} = 0.7$.

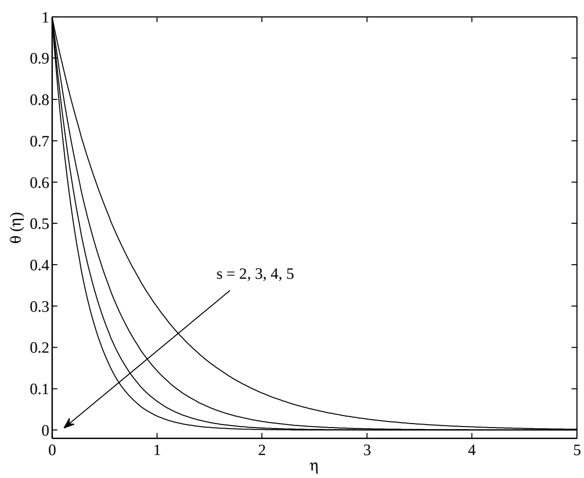

Fig. 7 Effects of the suction parameter s on the temperature profiles $\theta(\eta)$ when $\lambda = 2$, $M = 2$, $\mathrm{Pr} = 0.7$.

parameter λ on the temperature profiles $\theta(\eta)$ when $s = 3$, $M = 2$ and $\mathrm{Pr} = 0.7$, while Fig. 6 shows the influence of the magnetic parameter M when $s = 3$, $\lambda = 2$, and $\mathrm{Pr} = 0.7$. Both profiles show the reduction in boundary layer thickness as λ and M increase. However, the effects of these parameters are not very pronounced. Meanwhile, the effects of the suction parameter s when $M = 2$, $\lambda = 2$, $\mathrm{Pr} = 0.7$ and Prandtl number Pr when $s = 3$, $M = 2$, $\lambda = 2$ on the temperature profiles $\theta(\eta)$ are shown in Figs. 7 and 8. It can be seen that both profiles display the reduction in boundary layer thickness as s and Pr increase.

CONCLUSIONS

A numerical study was performed for the problem of MHD rotating flow and heat transfer due to a permeable shrinking sheet in a viscous fluid. This problem was solved numerically by using a Keller-

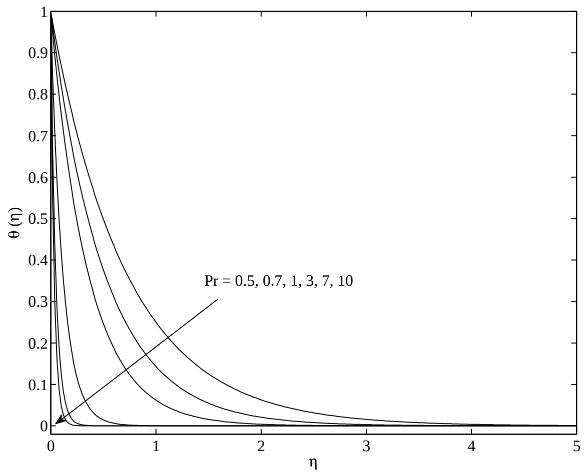

Fig. 8 Effects of the Prandtl number $\Pr$ on the temperature profiles $\theta(\eta)$ when $\lambda = 2$, $M = 2$, $s = 3$.

box method. The effects of the rotating parameter λ, magnetic parameter M, suction parameter s and Prandtl number $\Pr$ have been analysed and presented graphically. The results show that the boundary layer thickness for velocity profiles $f'(\eta)$ and $g(\eta)$ decrease with the increase of the rotating parameter λ, magnetic parameter M and suction parameter s. It is also found that the boundary layer thickness for temperature profiles $\theta(\eta)$ decrease with the increase of the rotating parameter λ, magnetic parameter M, suction parameter s and Prandtl number $\Pr$.

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