

Maximum likelihood estimation of linear structural relationship model parameters assuming the slope is known

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ABSTRACT: A number of statistical techniques have been proposed by many authors to estimate the parameters in a linear structural relationship model, but only few papers discuss the precision of these estimators. In this study, we derive the maximum likelihood estimate (MLE) of the parameters by assuming the slope parameter β is known. β is estimated separately by a nonparametric method and is assumed to be known when other parameters are estimated by an MLE. We obtain closed-form estimates of parameters as well as the variance-covariance matrix. Using a simulation study and a real-world example we show that the estimated values of the parameters are unbiased and consistent.

KEYWORDS: errors-in-variables model, nonparametric method, asymptotic variance-covariance matrix

INTRODUCTION

An errors-in-variables model (EIVM) is a study of relationships between variables, groups or populations in which errors are present in both the explanatory and response variables. It differs from the traditional regression where errors are present only in the dependent variable. Because of the measurement error in explanatory variables, the traditional regression often produces biased estimates of parameters. The linear structural relationship model (LSRM) is one of the families in the EIVM which also includes functional, ultrafunctional, and ultrastructural relationship models. Many authors have considered EIVM over the years¹⁻⁴. Although a lot of work has been done on the LSRM, only a few studies focused on the precision of the parameter estimation. Bolfarine and Cordani⁵, Patefield⁶ and Wong⁷ considered only a few special cases such as the accuracy of the slope and intercept. Hood et al⁸ first described the precision of the parameters and established the asymptotic variance-covariance matrix when the ratio of the two error variances is known. As for the estimation of slope parameter, several methods of estimation are proposed such as geometric mean⁹, by assuming equal error variance¹⁰, repeated median¹¹, and nonparametric

method¹².

As far as we know, there has been no attempt to estimate parameters assuming that the slope parameter β is known. Once we assume that β is known, we can easily estimate the other parameters by the maximum likelihood method. But the main advantage of our method is that, unlike other methods, β is not an arbitrary choice. In this article, we extend the idea of Al-Nasser¹² to estimate the slope parameter by a nonparametric method and then estimate the rest of LSRM parameters of by maximum likelihood estimation (MLE). The precision of the estimated parameters and its asymptotic variance-covariance matrix are also derived and investigated here through real world data and simulations.

The applications of EIVMs can be seen in studies in the life sciences such as biology, ecology, social science and economics where the variables cannot be recorded accurately. Since our proposed method of estimation is an improvement over the existing ones it should have wider applications in the above areas of research. This paper is organized in the following way. First, we briefly review different aspects of LSRM. Next, we propose a nonparametric method to estimate the slope parameter extending the work of Al-Nasser¹². The parameter estimation of LSRM by the

MLE method assuming the slope is given is proposed in the next section. Then we derive the asymptotic closed form of the variance-covariance information matrix. We also report on simulation studies which are designed to investigate the precision of the estimated parameters and its variance-covariance matrix. A real world example is presented finally to show the advantage of the proposed method over the existing ones.

LINEAR STRUCTURAL RELATIONSHIP MODEL

Consider the model

$$Y = \alpha + \beta X \tag{1}$$

where there exists a linear relationship between the random variables X and Y and suppose that they are measured without error. However, in reality they are subject to error. Assume that for each $i = 1, 2, \dots, n$, x_i and y_i are taken instead of X_i and Y_i , respectively. If δ_i and ϵ_i are the two respective errors in measuring X_i and Y_i , then we can write

$$x_i = X_i + \delta_i, \quad y_i = Y_i + \epsilon_i,$$

where the error terms δ_i and ϵ_i are normally distributed having zero mean and variance σ_δ^2 and σ_ϵ^2 , respectively. This reveals that the variances of error are not dependent on i and so independent of the level of X and Y . This property is known as homoscedasticity. There are some assumptions that have been described in the literature for obtaining the X values. For example, Kendall and Stuart¹³ described the structural model considering X_i as normal with mean μ and variance σ_X^2 . In LSRMs, the errors are assumed to be normal and the bivariate normal distribution of x_i and y_i is then

$$\begin{pmatrix} x_i \\ y_i \end{pmatrix} \sim N \left(\begin{pmatrix} \mu \\ \alpha + \beta\mu \end{pmatrix}, \begin{pmatrix} \sigma_X^2 + \sigma_\delta^2 & \beta\sigma_X^2 \\ \beta\sigma_X^2 & \beta^2\sigma_X^2 + \sigma_\epsilon^2 \end{pmatrix} \right)$$

Kendall and Stuart¹³ stated the moment equations for the parameters of LSRM in the form

$$\begin{aligned} \mu &= \bar{x}, & \beta\mu + \alpha &= \bar{y}, \\ \sigma_X^2 + \sigma_\delta^2 &= S_x^2, & \beta^2\sigma_X^2 + \sigma_\epsilon^2 &= S_y^2, & \beta\sigma_X^2 &= S_{xy}, \end{aligned}$$

where $S_x^2 = (1/n) \sum (x_i - \bar{x})^2$, $S_y^2 = (1/n) \sum (y_i - \bar{y})^2$, and $S_{xy} = (1/n) \sum (x_i - \bar{x}) \sum (y_i - \bar{y})$. Clearly there are five equations in six unknowns, so some additional assumptions have to be made about the parameters of the LSRM in order to overcome this unidentifiability problem. Hood et al⁸ considered the following five cases for unique and consistent solutions to the equations.

- (i) both the error variances σ_δ^2 and σ_ϵ^2 are known;
- (ii) σ_δ^2 is known;
- (iii) σ_ϵ^2 is known;
- (iv) both the error variances σ_δ^2 and σ_ϵ^2 are unknown;
- (v) the error variances ratio $\lambda = \sigma_\epsilon^2/\sigma_\delta^2$ is known;

In this article, we develop another new case to solve the unidentifiability problem of estimating parameters in LSRMs. We first estimate the slope parameter by modifying the nonparametric method and then the rest of the parameters of the LSRM are estimated by the maximum likelihood method.

ESTIMATION OF SLOPE PARAMETER BY USING NONPARAMETRIC METHOD

In this method, the observed pairs (x_i, y_i) , ($i = 1, 2, \dots, n$) are ordered such that $x_i < x_{i+1}$ assuming all the x_i are distinct. Then we distribute the observations into several groups to calculate all possible paired slopes. Furthermore, another possible pair of slopes is determined by ordering the observed pairs according to the magnitude of the y value. The complete procedure is summarized in the following algorithm.

Algorithm 1

- Step 1: We arrange the observations in ascending order on the magnitude of x i.e., $x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(n)}$. We take the associated values of y , i.e., $y_{[1]}, y_{[2]}, \dots, y_{[n]}$ which may not be in ascending order and obtain our desire pairs $(x_{(i)}, y_{[i]})$.
- Step 2: We distribute the n observations into m subsamples each having r elements such that $mr = n$.
- Step 3: Find the number of all possible combination of paired slopes $b^{(k)}_{ij} = (y_{[j]} - y_{[i]}) / (x_{(j)} - x_{(i)})$, where $i = 1, 2, \dots, j - 1$; $j = 2, 3, \dots, r$; and $k = 1, 2, \dots, m$.
- Step 4: Calculate $\beta_x = \text{median}_k b^{(k)}_{ij}$.
- Step 5: We repeat Steps 1–4 by interchanging y and x . Finally, we define the estimated value of the slope as $\hat{\beta}_{\text{new}} = \frac{1}{2}(\beta_x + \beta_y)$.

Steps 1–4 have been proposed by Al-Nasser¹² and to improve the estimate we propose Step 5 by virtue of the symmetrical nature of variables X and Y in EIVM.

MLE OF A LSRM ASSUMING KNOWN SLOPE

Once the slope is estimated using the nonparametric method as described in the previous section, the other parameters can be estimated using the maximum likelihood method for the given value of the slope parameter. In this section, we briefly describe the procedure

of deriving the estimating equation for the LSRM by using MLE. One advantage of this approach is that it is easy to get the asymptotic variance-covariance matrix of the estimators. Let

$$A = S_x^2(\beta^2\sigma_X^2 + \sigma_\epsilon^2) - 2S_{xy}\beta\sigma_X^2 + S_y^2(\sigma_X^2 + \sigma_\delta^2),$$

the determinant of the variance-covariance matrix is

$$B = \beta^2\sigma_X^2\sigma_\delta^2 + \sigma_\delta^2\sigma_\epsilon^2 + \sigma_X^2\sigma_\epsilon^2$$

and

$$C = (\bar{x} - \mu)^2(\beta^2\sigma_X^2 + \sigma_\epsilon^2) - 2\beta\sigma_X^2(\bar{x} - \mu)(\bar{y} - \alpha - \beta\mu) + (\bar{y} - \alpha - \beta\mu)^2(\sigma_X^2 + \sigma_\delta^2).$$

The log-likelihood function can be rewritten in the following way

$$l = -n \log(2\pi) - \frac{n}{2} \left\{ \frac{A}{B} + \frac{C}{B} + \log(B) \right\}. \quad (2)$$

The term C in log-likelihood function will be minimized, and equal to zero, when $\mu = \bar{x}$ and $\beta\mu + \alpha = \bar{y}$. From this equation, maximum likelihood estimates of μ and α can be determined. Since the terms A and B in the log-likelihood function do not contain the parameters μ and α , the likelihood function is then solved by minimizing

$$G = -\frac{n}{2} \log(B) - \frac{n}{2} \left(\frac{A}{B} \right)$$

with respect to σ_X^2 , σ_δ^2 and σ_ϵ^2 . The partial derivatives are taken and set equal to zero to give

$$\begin{aligned} (\beta^2\sigma_\delta^2 + \sigma_\epsilon^2 + \beta^2S_x^2 - 2\beta S_{xy} + S_y^2)B \\ - (\beta^2\sigma_\delta^2 + \sigma_\epsilon^2)A &= 0 \\ (\sigma_\epsilon^2 + \beta^2\sigma_X^2 + S_y^2)B - (\sigma_\epsilon^2 + \beta^2\sigma_X^2)A &= 0 \\ (\sigma_\delta^2 + \sigma_X^2 + S_x^2)B - (\sigma_\delta^2 + \sigma_X^2)A &= 0. \end{aligned} \quad (3)$$

We solve the above three equations and get

$$\begin{aligned} \sigma_X^2 &= \frac{S_{xy}}{\beta}, \quad \sigma_\delta^2 = \frac{(\beta S_x^2 - S_{xy})}{\beta}, \\ \sigma_\epsilon^2 &= (S_y^2 - \beta S_{xy}). \end{aligned} \quad (4)$$

Admissible solutions are obtained when $\beta S_x^2 > S_{xy}$ and $S_y^2 > \beta S_{xy}$. For some specific data set, if these criteria are not satisfied, we then set one or more variances equal to zero in order to find the maximum likelihood estimators for the parameters. Hence, if the first and second conditions are broken then following Ref. 8, we set $\sigma_\delta^2 = 0$ and $\sigma_\epsilon^2 = 0$.

VARIANCE-COVARIANCE MATRIX OF A LSRM

The Fisher information matrix (I_{ij}) for the set of unknown parameters θ is given by

$$(I_{ij}) = \left[-E \left(\frac{\partial^2 l}{\partial \theta_i \partial \theta_j} \right) \right].$$

The above information matrix is inverted in order to find the asymptotic variance-covariance matrix. From (2),

$$l = -n \log(2\pi) - \frac{n}{2} \log(B) - \frac{D}{2B}$$

where

$$D = (\beta^2\sigma_X^2 + \sigma_\epsilon^2) \sum (x_i - \mu)^2 - 2\beta\sigma_X^2 \sum (x_i - \mu)(y_i - \alpha - \beta\mu) + (\sigma_X^2 + \sigma_\delta^2) \sum (y_i - \alpha - \beta\mu)^2.$$

After lengthy algebraic manipulations, the inverse of the Fisher information matrix is given by

$$\frac{1}{n} \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ & a_{22} & a_{23} & a_{24} & a_{25} \\ & & a_{33} & a_{34} & a_{35} \\ & & & a_{44} & a_{45} \\ & & & & a_{55} \end{pmatrix}$$

where $a_{11} = \sigma_X^2 + \sigma_\delta^2$, $a_{12} = -\beta\sigma_\delta^2$, $a_{13} = 0$, $a_{14} = 0$, $a_{15} = 0$, $a_{22} = \beta^2\sigma_\delta^2 + \sigma_\epsilon^2$, $a_{23} = 0$, $a_{24} = 0$, $a_{25} = 0$, $a_{33} = 2\sigma_X^4 + B/\beta^2$, $a_{34} = 2\sigma_\delta^2\sigma_X^2 - B/\beta^2$, $a_{35} = 2\sigma_\epsilon^2\sigma_X^2 - B$, $a_{44} = 2\sigma_\delta^4 + B/\beta^2$, $a_{45} = 2\sigma_\epsilon^2\sigma_\delta^2 - B$, $a_{55} = 2\sigma_\epsilon^4 + \beta^2 B$.

The above results clearly show that the variances and covariances of the estimated parameters converge to 0 as n tends to infinity. Hence all parameters are consistent.

SIMULATION STUDY

A simulation study was carried out to support the algebraic results presented earlier. For simulation experiments, we considered the various parameter settings given in Table 1. In selecting σ_X^2 we follow the principle of Ref. 8 in which the difference caused by measurement errors will probably be dominated by the difference between the mean levels. For each specified set of parameter values, 15 000 simulated data sets were obtained for the sample sizes n in Table 1. The performance of the estimated parameters are measured by estimated bias (EB), estimated root mean square error (ERMSE) and standard deviation (SD) where $EB = |\hat{w} - w|$ and $ERMSE = \sqrt{(1/s) \sum (\hat{w}_j - w)^2}$ where w is a generic term for

Table 1 Results for $\alpha = 0, \mu = 10, \sigma_X^2 = 5$ with 3 different sets of $(\beta, \sigma_\delta, \sigma_\epsilon)$. n is the sample size.

Statistic	n	0.80,0.90,0.7(5)					(1.00,0.75,1.00)					(1.20,1.00,0.80)				
		100μ	α	β	σ_X^2	σ_δ^2	100μ	α	β	σ_X^2	σ_δ^2	100μ	α	β	σ_X^2	σ_δ^2
EB	50	0.102	0.0033	0.098	0.0128	0.0184	0.150	0.0020	0.110	0.0157	0.0239	0.186	0.0017	0.105	0.0182	0.0215
	70	0.051	0.0024	0.056	0.0112	0.0132	0.092	0.0015	0.082	0.0120	0.0153	0.041	0.0012	0.074	0.0166	0.0142
	100	0.021	0.0005	0.048	0.0082	0.0075	0.030	0.0011	0.048	0.0057	0.0138	0.036	0.0010	0.043	0.0065	0.0088
	200	0.011	0.0003	0.028	0.0068	0.0042	0.025	0.0008	0.024	0.0045	0.0079	0.024	0.0008	0.018	0.0063	0.0060
	300	0.004	0.0001	0.014	0.0043	0.0027	0.015	0.0006	0.016	0.0031	0.0024	0.002	0.0006	0.013	0.0058	0.0045
ERMSE	500	0.002	0.0000	0.003	0.0017	0.0011	0.001	0.0001	0.012	0.0010	0.0016	0.001	0.0002	0.007	0.0002	0.0029
	50	16.322	0.3441	1.108	0.5054	0.3378	18.742	0.3416	1.091	0.4530	0.4711	21.090	0.3440	1.086	0.4468	0.6051
	70	13.843	0.2890	0.920	0.4249	0.2849	15.560	0.2864	0.921	0.3847	0.3971	17.821	0.2935	0.912	0.3836	0.5110
	100	11.524	0.2439	0.771	0.3654	0.2421	13.171	0.2411	0.775	0.3298	0.3408	14.815	0.2420	0.765	0.3193	0.4271
	200	9.301	0.0813	0.548	0.2557	0.1715	10.921	0.1240	0.542	0.2294	0.2365	11.303	0.1864	0.546	0.2246	0.3020
SD	300	5.783	0.0667	0.449	0.2057	0.1387	8.799	0.0760	0.451	0.1868	0.1961	9.248	0.1270	0.438	0.1857	0.2489
	500	0.930	0.0515	0.352	0.1599	0.1065	3.803	0.0586	0.344	0.1453	0.1512	3.955	0.0868	0.341	0.1438	0.1934
	50	16.129	0.3402	1.086	0.5008	0.3335	18.493	0.3353	1.067	0.4507	0.4687	20.932	0.3428	1.057	0.4451	0.5960
	70	13.663	0.2887	0.926	0.4262	0.2839	15.689	0.2842	0.907	0.3833	0.3988	17.752	0.2905	0.899	0.3784	0.5074
	100	11.457	0.2417	0.776	0.3575	0.2384	13.156	0.2386	0.764	0.3226	0.3353	14.882	0.2439	0.757	0.3185	0.4263
RM	200	10.021	0.1811	0.551	0.2536	0.1694	6.922	0.1233	0.543	0.2295	0.2385	9.082	0.1555	0.538	0.2262	0.3034
	300	9.810	0.1064	0.451	0.2079	0.1387	4.825	0.0762	0.444	0.1877	0.1952	6.117	0.0962	0.439	0.1849	0.2480
	500	5.601	0.0814	0.350	0.1611	0.1074	2.314	0.0356	0.344	0.1353	0.1312	1.257	0.0468	0.341	0.1035	0.1623
	50	1.663	0.0192	1.576	1.4605	0.7183	0.367	0.0024	1.955	1.8511	1.0099	1.277	0.0031	2.398	1.0024	2.7959
	70	4.992	0.0051	1.414	0.8570	0.6991	1.224	0.0102	2.524	1.0834	0.7976	1.343	0.0279	2.109	0.5985	2.6316
100	0.905	0.0111	1.490	0.8325	0.6069	2.022	0.0010	1.383	1.0844	0.5668	1.100	0.0046	1.183	0.6714	1.2388	
200	4.950	0.0826	1.651	0.5988	0.3620	5.575	0.0009	0.745	0.4364	0.2840	10.744	0.1996	1.673	0.6524	0.4864	
300	17.977	0.0947	0.909	0.6316	0.3417	3.701	0.0150	1.423	0.2456	0.7808	2.811	0.0054	1.302	0.7127	0.4937	
500	19.141	0.1091	1.317	0.2878	0.2654	6.614	0.2964	0.924	0.3019	0.1103	3.253	0.0575	2.345	0.3223	0.2110	

the parameters and SD for each parameter are computed from the diagonal element of the asymptotic variance-covariance matrix. We also tried to investigate whether the distribution of the estimators are asymptotically normal or not. For testing normality of the distribution of the estimators we have used the rescaled moment (RM) test proposed by Imon¹⁴ which is an extension of the Jarque-Bera normality test. We have used the RM test here because the Jarque-Bera test has a well known disadvantage of possessing poor power. When we denote the sample skewness and sample kurtosis of the estimators by S and K , respectively, then the RM value is defined as

$$RM = \frac{nc^2}{6} \left[S^2 + \frac{(K-3)^2}{4} \right]$$

where n is the sample size and $c = n/(n-k)$ when k is the number of parameters. The distribution of RM follows a χ^2 distribution with 2 degrees of freedom.

For each specified set of parameters in Table 1, the estimated bias (EB) of all parameters are close to 0 and become smaller when the sample size increases, which shows the unbiasedness of the parameters. From Table 1, the relative EB for σ_X^2 is 1.966 for $n = 50$ and it reduces to 0.058 for $n = 500$. Similarly, the relative EB for σ_δ^2 decreases from 1.422 for $n = 50$ to 0.189 for $n = 500$. Also, the decrease in relative EB for σ_ϵ^2 is seen from 2.453 for $n = 50$ to 0.147 for $n = 500$. Similar remarks may apply with other measurements such as ERMSE and SD where these values tend to decrease with increase in sample sizes. These results clearly suggest that the estimated values of parameters are consistent. We

also observe that each of the estimated parameters individually follows a normal distribution. Here, the cut-off value for the RM statistic is 5.99 but all RM values we obtain from this experiment are much less than this cut-off value.

NUMERICAL EXAMPLE

In this section we consider a real world data to estimate the parameter of this model. In order to make the relationship as model (1), we assume that measurement error can occur in both the dependent and independent variables of this data set.

Here we consider the serum kanamycin data taken from Kelly¹⁵. In order to measure the serum kanamycin levels in blood samples, simultaneous pairs of measurements were taken from twenty premature babies. These two measurements were obtained by the heel stick method (independent variable) and the umbilical catheter method (dependent variable).

We consider five different cases to estimate parameters and their standard deviation for serum kanamycin data and compared those with the results obtained by the proposed method. It is worth mentioning that case 4 is computationally impossible for any real data. The results are given in Table 2

Results presented in Table 2 give a clear indication of using the proposed method for estimating parameters and their standard deviations in a LSRM. Case 4 is impossible to compute. Out of 6 parameters we cannot estimate two parameters for case 1 and one parameter for case 2, case 3, and case 5. Only the proposed method can estimate all 6 parameters. In terms of precision we observe that for μ our proposed

Table 2 Comparison of estimated parameters (standard deviations in parentheses) of different MLE methods for serum kanamycin data.

Case	μ	α	β	σ_X^2	σ_δ^2	σ_ϵ^2
1	20.9 (1.5)	-1.2 (1.1)	1.1 (0.1)	21.9 (6.9)	-	-
2	20.9 (1.1)	2.0 (7.3)	0.9 (0.2)	23.7 (7.8)	-	26.7 (8.7)
3	20.9 (1.1)	-4.4 (274.9)	1.2 (4.5)	0.9 (282.9)	23.9 (7.8)	-
5	20.9 (1.1)	-1.2 (3.3)	1.1 (0.2)	20.4 (7.8)	4.4 (1.4)	-
Proposed	20.9 (1.1)	-0.1 (0.7)	1.0	21.3 (7.5)	3.4 (3.4)	5.4 (3.7)

method has better precision than case 1, however it has the same precision as cases 2, 3, and 5. For α the proposed method outperforms all four cases. This method yields lower standard deviations than all other cases. For σ_X^2 the proposed method gives smaller standard deviation than cases 2, 3, and 5 however its standard deviation is slightly more than that of case 1. For σ_δ^2 and σ_ϵ^2 the proposed method clearly outperforms the other estimation methods. Thus the proposed method performs best overall.

CONCLUSIONS

In this article, a new maximum likelihood method is proposed to estimate parameters of a linear structural relationship model based on the assumption of known slope. We then derive the asymptotic variance-covariance matrix of the parameters in simple form. With the help of simulation study we have showed that the estimated values of the parameters are unbiased and consistent even for relatively small samples. Real world data shows that the proposed method has better precisions in comparison with the existing MLE alternatives.

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