A MICROSCOPIC THEORY FOR THE INTERLAYER ENHANCEMENT OF SUPERCONDUCTIVITY IN THE HTSC'S

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ABSTRACT

The effects of interlayer interaction through the particle-particle channel and the particle-hole channel on an electron-phonon (breathing mode) model of the high T_c superconductor are studied. The strong coupling Eliashberg formalism is used in the study. It is shown that the interaction through the former channel can appreciably enhance the T_c of the two CuO_2 layer superconductor ($YBa_2Cu_3O_7$) over that of the single layer superconductor ($La_{2-x}(Sr,Ba)_xCuO_4$). The calculations are done in the weak coupling limit (the $\lambda^{\theta\theta}$ model) of the Eliashberg formalism.

INTRODUCTION

The discovery of superconductivity in the doped fullerenes [1] has created much excitement in the field since pair condensation in the fullerene was so unexpected. The highest T_c achieved, that of $Cs_xRb_yC_{80}$, [2] is comparable to the T_c of the first cuprate high T_c superconductor discovered, $La_{2-x}Ba_xCuO_4$ [3] (33 K versus 35 K). While the isotope effect in $La_{2-x}Ba_xCuO_4$ is greatly reduced, [4] it is quite large in the fullerene superconductor, Rb_3C_{80}. Ebbesen *et al.*, [5] obtained an isotope shift exponent of 1.4 ± 0.5. This value and somewhat smaller values obtained by others point to the electron-phonon interaction as being the dominant mechanism for superconductivity in the doped fullerene. In fact, it has been established that the active phonons are the higher frequency intramolecular H_g modes. [6, 7]

In one of the earlier theories [8, 9] for superconductivity in the cuprate high T_c superconductors (HTSC), it was proposed that the pairing was via the high frequency oxygen breathing mode vibration. Weber [8] studied the el-ph interactions in $La_{2-x}(Ba, Sr)_xCuO_4$ and concluded that the high T_c's of these superconductors were not caused by the unusually strong el-ph coupling, but were caused by the fact that the 2F( ), the spectral density function, extends to much higher frequencies than in the case for typical transition-metal superconductors. Weber and Mattheiss [9] studied the el-ph interaction in the $YBa_2Cu_3O_7$ and found that the el-ph coupling would not be strong enough to account for a T_c of 90 K. They closed their paper with the remark, "An additional or alternative mechanism for electronic pairing is required."

Insertion of additional CuO_2 layers into the bismuth and thallium superconductors increased the T_c's of these superconductors. [10, 11] Using the Ginzburg Landau approach, two groups showed that these increases in the T_c's could be accounted for by an interlayer coupling of the order parameters belonging to adjacent layers [12, 13]. Eab and Tang [12] took the interlayer coupling to be due to a nearest layer bilinear Josephson coupling, while Birman and Lu [13] took the coupling to be due to some type of nearest layer spin-spin interaction. Recently, clear evidences for Josephson tunneling between CuO_2 layers has been seen in the single crystal superconductors, $Bi_2Sr_2CaCu_2O_8$, $(Pb_yBi_{1-y})_2Sr_2CaCu_2O_8$ and $Tl_2Ba_2Ca_2Cu_3O_{10}$ [14]. The magnetic field dependence of the c-axis critical currents in the bismuth superconductors pointed to Josephson pair tunneling between superconducting layers (formed by the double CuO_2 layers) separated by 15 A thick insulating layers (formed by the Bi_2O_3 and SrO layers). Microwave emission measurements indicate that these three HTSC's could be considered to be stacks of intrinsic SIS Josephson junctions. Evidences for Josephson tunneling in other single crystal HTSC's, $YBa_2Cu_4O_8$ and $Nd_{1.85}Ce_{0.15}C_2O_{4-y}$, have been seen by Basov *et al.*, [15] and by Zuo *et al.*, [16] respectively.

In one of the first microscopic studies on these interactions in the high T_c superconductors, Tesanovic [17] showed that the interlayer interaction in the particle-particle channel could enhance the T_c. The purpose of this paper is to examine the effects of the interlayer coupling on the HTSC within the framework of the strong coupling approach. It is assumed that the primary mechanism for superconductivity is the el-ph mechanism with the active phonon being the high frequency oxygen breathing mode.

ELIASHBERG EQUATIONS

Since most evidences point to the cuprate high T_c superconductors being strong coupling superconductor, the Eliashberg formalism should be used to study these superconductors. To begin, we first note that the coupling can be through either the particle-hole channel or the perticle-particle channel.[17] The following perturbations

$$t_1 \sum_{k, k'; \sigma} C^*_{k,\sigma,i}\, C^*_{-k,-\sigma,j}\, C_{k',-\sigma,j}\, C_{-k',\sigma,i} \qquad (1)$$

and

$$t_2 \sum_{k, k'; \sigma} C^*_{k,\sigma,i}\, C^*_{-k,-\sigma,i}\, C_{k',-\sigma,j}\, C_{-k',\sigma,j} \qquad (2)$$

represent the interactions through the two channels, respectively. In the above, t_1 and t_2 are the two interlayer coupling strengths; k and k' , the momentums; σ, spin labels and i and j label adjacent layers. These interactions are assumed not to have originated from the direct hopping of an electron from one layer to another. Instead, it was suggested in ref. 17, that the interaction in the particle-hole channel arises from either from plasmon, exciton or phonon assisted transitions and that the interaction in the particle-particle channel generates "Josephson-like tunneling" coupling of order parameters belonging to adjacent layers.

The screened interaction appearing in Fig. 1a, is generated by the particle-hole channel interaction, while the screened interaction appearing in Fig. 1b, is generated by the particle-particle channel interaction. The bubbles in the screened interactions are the two particle propagators with different vertices. The diagrams in Figs. 1a and 1b represent the diagonal and off diagonal parts of the self energy corrections due to the interlayer coupling. These should be included with the corrections arising from the el-ph interaction and from the Coulomb interaction. When done, the following Eliashberg equations are obtained

$$\{Z(i\omega_n)-1\}\omega_n = \pi k_B T \sum_m \lambda(n\text{-}m) \frac{\omega_m}{[\omega_m^2 + |\Delta_m|^2]^{1/2}}$$

$$+ \pi k_B T \sum_m T_1 \frac{\omega_m}{[\omega_m^2 + |\Delta_m|^2]^{1/2}} \qquad (3)$$

and

$$Z(i\omega_n)\Delta_n = \pi k_B T \sum_m (\lambda(n\text{-}m) - \mu^* \theta(\omega_{cutoff} - |\omega_m|)) \frac{\Delta_m}{[\omega_m^2 + |\Delta_m|^2]^{1/2}}$$

$$+ k_B T \sum_m T_2 \frac{\Delta_m}{[\omega_m^2 + |\Delta_m|^2]^{1/2}} \qquad (4)$$

where $\omega_m = i\pi k_B T(2m+1)$. μ^* is the Coulomb pseudo potential defined by

$$\mu^* = \mu / \{ 1 + \mu \ln(\omega_{cutoff}/\pi T_c) \} \qquad (5)$$

with the cutoff frequency being the breathing mode frequency for the case of HTSC. The el-ph pairing parameter is given by

$$\lambda(n\text{-}m) = 2\int \frac{\Omega\, d\Omega\, \alpha^2 F(\Omega)}{\Omega^2 + (\omega_m - \omega_n)^2} \qquad (6)$$

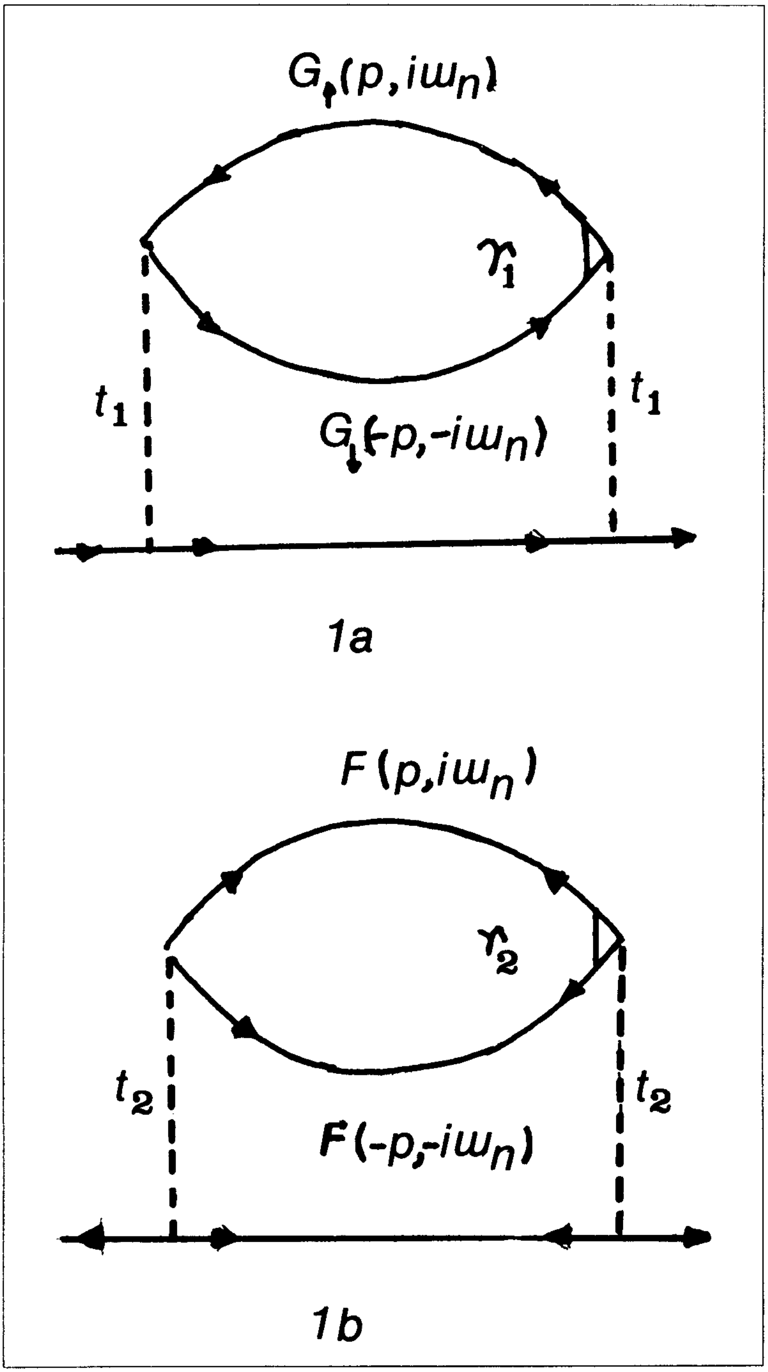

Fig. 1. Self Energy Correction. 1a. Self energy correction due to the interlayer coupling in the particle-hole channe. 1b. Correction due to the interlayer coupling in the particle-particle channel.

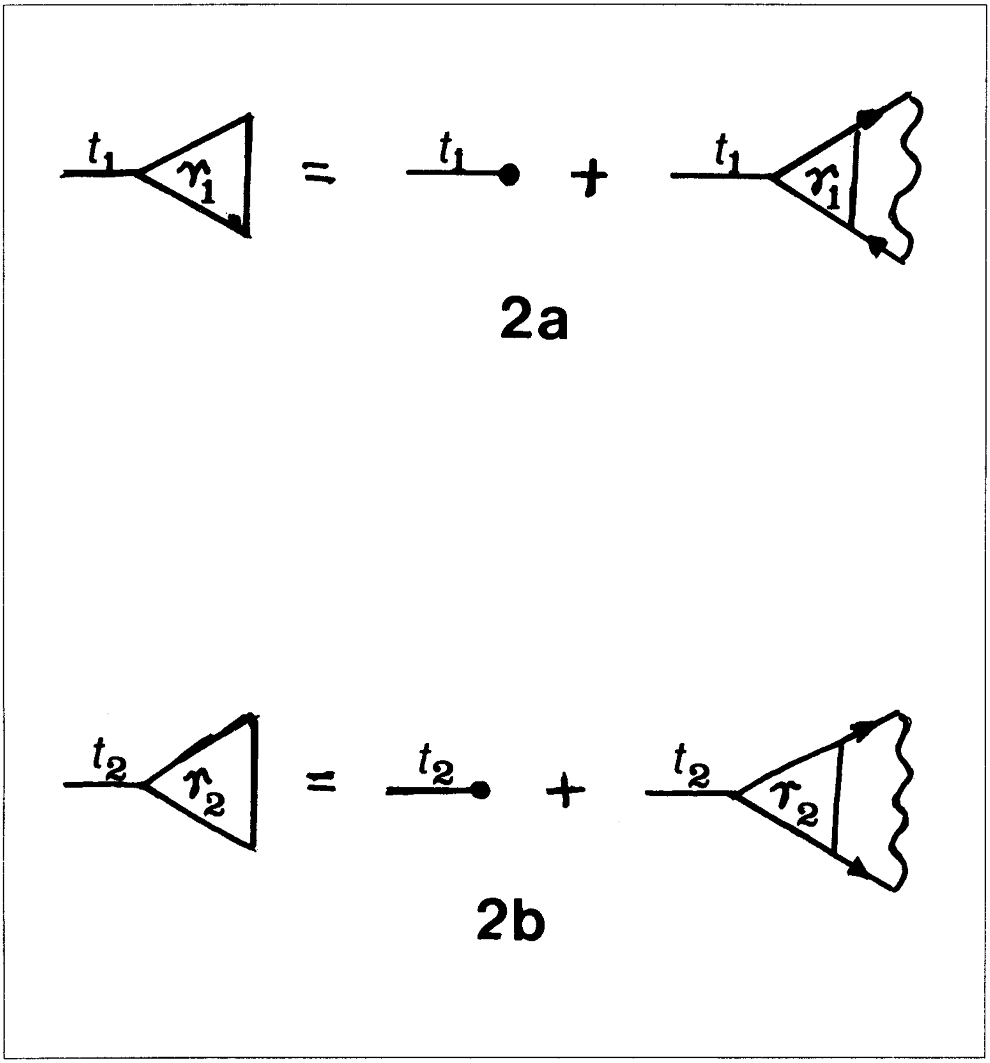

Fig. 2. Vertex Correction. 1a. The Bethe-Salpeter equation for the vertex appearing in the bubble diagram appearing in the self energy correction defined by Fig. 1a. 1b. The Bethe-Salpeter equation for the vertex appearing in the bubble diagram occuring in Fig. 1b. It is assumed that the main correction to the vertex is the exchange of a phonon.

with $\alpha^2F(\Omega)$ being the spectral density function for the superconductor. These functions for $La_{2-x}Ba_xCuO_4$ and $YBa_2Cu_3O_7$ have been calculated theoretically by Weber [8] and Weber and Matheiss, [9] respectively, while the spectral density function for $Nd_{2-x}Ce_xCuO_4$ has been determined experimentally by Huang *et al.* [18] T_1 and T_2 are the interlayer ùcoupling energies' defined as

$$T_1 = \pi k_B T \sum_m \int d^3p\, t_1^2\, G(p, i\omega_m)\, G(-p, -i\omega_m)\, \gamma_1(i\omega_m) \qquad (6)$$

and

$$T_2 = \pi k_B T \sum_m \int d^3p\, t_2^2\, F(p, i\omega_m)\, F(-p, -i\omega_m)\, \gamma_2(i\omega_m) \qquad (7)$$

where γ_1 and γ_2 are the vertices defined by Figs. 2a and 2b, respectively. $G(p, i\omega_m)$ and $F(p, i\omega_m)$ are the single paricle propagators.

T_C ENHANCEMENT

Combining the Eliashberg equations together, we get

$$\Delta_n \{ 1 + \pi k_B T \sum_{m=-\infty}^{\infty} [\lambda(n-m) + T_1] \} \frac{\Delta_m}{[\omega_m^2 + |\Delta_m|^2]^{1/2}} =$$

$$\pi k_B T \sum_{m=-\infty}^{\infty} [\lambda(n-m) - \mu^* \theta(w_{cutoff} - |\omega_m|) + T_2] \frac{\Delta_m}{[\omega_m^2 + |\Delta_m|^2]^{1/2}} \qquad (8)$$

The simplest solution is achieved in the square well $\lambda^{\theta\theta}$ model (the weak coupling limit) where the el-ph pairing parameter $\lambda(n-m)$ is approximated by

$$\lambda(n-m) = \theta(\omega_{cutoff} - |\omega_n|)\, \lambda\theta(\omega_{cutoff} - |\omega_m|)$$

with $\lambda = \lambda(0)$. With this assumption, it follows that the order parameter Δ_n is independent of ω_n, i.e., the order parameter is a constant. We will also assume that T_1 and T_2 are independent of ω_n. Working in the weak coupling limit of Eliasberg approach, we get

$$T_C \sim \omega_{cutoff} \exp - \{ (1 + \lambda + T_1) / (\lambda - \mu^* + T_2) \} \qquad (10)$$

where ω_{cutoff} is the highest oxygen breathing mode frequency and λ is the el-ph coupling strength for the HTSC.

As we have pointed out, Weber [8] has calculated the spectral density function $\alpha^2(\Omega)$ F (Ω) for $La_{2-x}(Sr, Ba)_xCuO_4$. He found that the function extends up to frequencies of 80 meV or so. The higher frequencies arise from the O(1) and O(2) stretching modes. Weber calculated the el-ph coupling strength for the La superconductor and found it to be in the range 2 to 2.5, similar to the values of for the A15 compounds, which have low T_c's. Using a value of 0.13 for the Coulomb pseudo potential μ^*, a T_c of between 30 and 40 K was predicted using the standard T_c formula [19]

$$T_c = h\omega_{cutoff} \exp - \{ (1+\lambda) / (\lambda - \mu^*) \} \tag{11}$$

Performing a similar calculation, Weber and Matheiss [9] obtained the spectral density function for $YBa_2Cu_3O_7$. They found the el-ph coupling strength for $YBa_2Cu_3O_7$ superconductor to be between 1.3 and 2.3. Using the same value the Coulomb pseudo potential μ^*, a T_c of between 19 and 30 K was predicted for $YBa_2Cu_3O_7$. The lower T_c is also due in part to the fact that the spectral density function extends up to 70 meV only. Weber and Mattheiss point out that increasing the value of λ could not lead to the needed increase in T_c since the presence of λ in both the numerator and denominator of the exponential factor in eqn. (11), causes a saturation of the T_c values. They concluded that the jump in the T_c of $YBa_2Cu_3O_7$ from 19-30 K to 90 K would have to be due to the presence of an additional mechanism in $YBa_2Cu_3O_7$ which is not present in $La_{2-x}Ba_xCuO_4$.

The expression for the transition temperature given by eqn.(11) does not contain any parameters which are explicitly connected to the main difference between the two HTSC's, i.e., the presence of a double CuO_2 layer in $YBa_2Cu_3O_7$ as oppose to only a single CuO_2 layer in $La_{2-x}(Sr, Ba)_xCuO_4$. Equation 10, however, does. To bring out the difference between eqns. (10) and (11), we combine the two to obtain

$$T_c = T_c^o \exp \{ (1+\lambda) / (\lambda - \mu^*) - (1+\lambda+T_1) / (\lambda - \mu^* + T_2) \} \tag{12}$$

where T_c^o is transition temperature in the absence of the interlayer interactions, i.e., the transition temperature given by eqn. (11). Figures 3a and 3b show the plots of Tc versus the interlayer coupling energies T2 for different values of T_1. The value of the Coulomb pseudo potential is set at 0.13. In Fig. 3a, we plot the dependence of Tc on T_2 when the el-ph coupling strength is set at 1.3 and 2.3, the lower and upper values of the el-ph coupling strength given by Weber and Mattheiss [9] for $YBa_2Cu_3O_7$ and T_1 is set to zero. As we see, T_2, the interlayer energy arising from the coupling in the particle-particle channel, can enhance the T_c appreciably, the effect being more evident for smaller values of λ. In Figure 3b, we plot the dependence of T_c for different values of T_1, the interlayer energy arising from the coupling in the particle-hole channel. The value of el-ph couling strength used 1.3. As we see, T_1 causes a lowering of the transition temperatures.

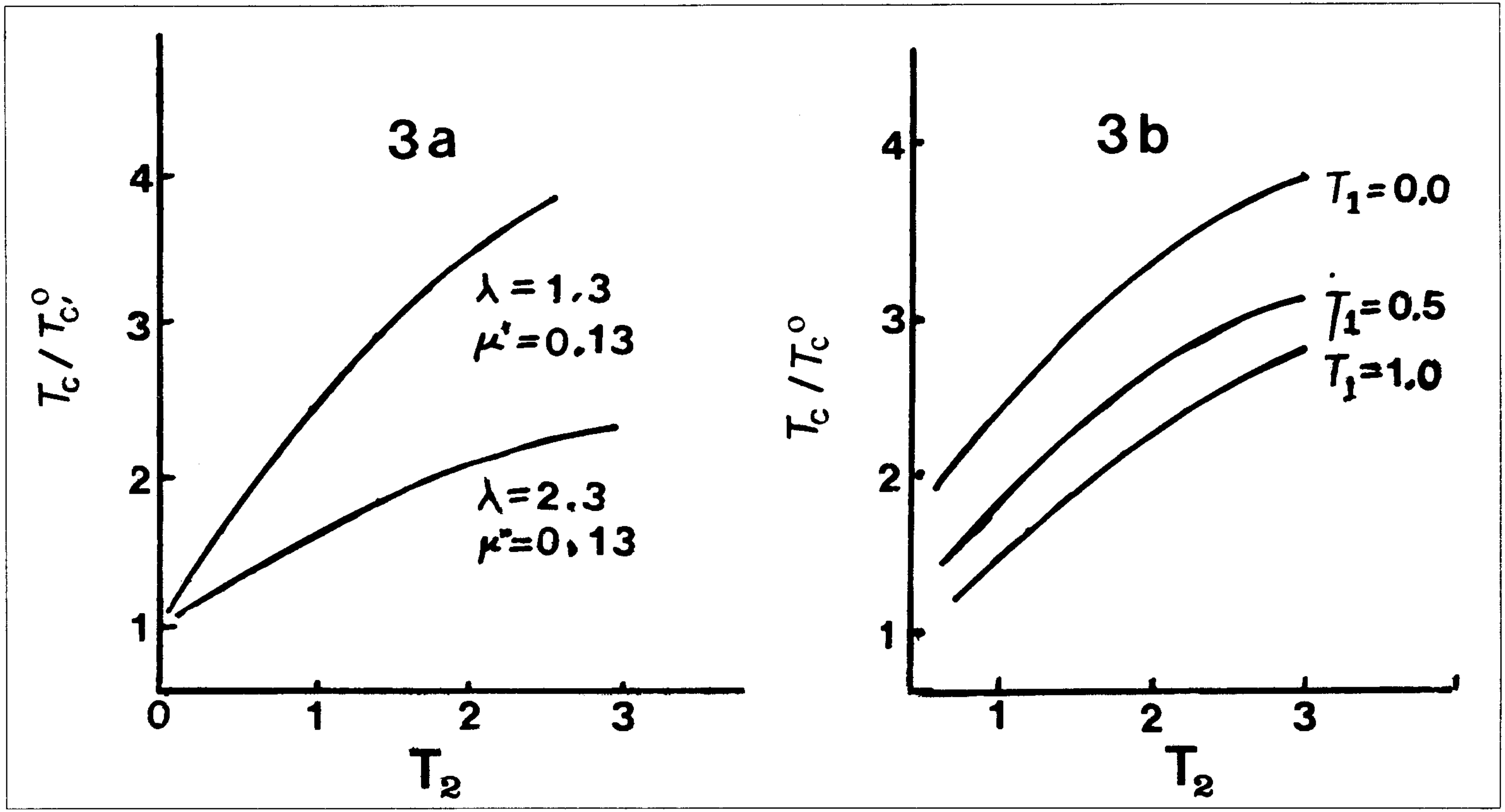

Fig. 3. Dependence Of Tc. On the Interlayer Interaction T_1. and T_2.. 3a. Dependence of Tc. on the interlayer interaction via the particle-particle channel, T_2.. The two values of λ are the upper and lower values of the electron-phonon strengths given by Weber and Mattheiss. [9] 3b. Effect of interlayer interaction in the particle-hole channel, T_1, on Tc.

Egn. 12 and the dependence of the enhancement of T_c on the value of interlayer interaction strength T_2 exhibited by Fig. 3a may also provide a possible framework for understanding the differences in the values of the $T_{c's}$ of all two copper layer HTSC's containing the $CuO_2/Ca/CuO_2$ layer. The strength of the Josephson coupling across a barrier of thickness d goes as d^{-2}, meaning that as the distance between the CuO_2 layers decreases, the Josephson coupling strength should increase leading to a higher T_c. Harshman and Mills [20] considered all novel superconductors in their attempt to find correlations between the various parameters of the superconductors. They found the Tc's of most of the members of the above set of HTSC's correlates with d^{-1} (See Fig. 12b of ref. 20). The dependence of T_c on the distance between the CuO_2 layers implied by eqn. (12) would be much more complicated than a simple d^{-1} or d^{-2} dependence since other parameters such as the breathing mode frequency and the el-ph coupling stengths may also depend on d. Nevertheless, the increased enhancement of T_c seen in Figs. 3a and 3b is consistent with the correlation observed in ref. 20.

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REFERENCES

1. A.F. Hebard *et al.*, *Nature* **350** (1991) 600; K. Holczer *et al.*, *Science* **252** (1991) 1154.
2. K. Tanigaki *et al.*, *Nature* **302** (1991) 222.
3. J.G. Bednorz and K.A. Muller, *Z. Phys.* B **64** (1986) 189.
4. B. Batlogg *et al.*, *Phys. Rev. Lett.* **59** (1987) 912.
5. T.W. Ebbesen *et al.*, *Nature* **355** (1992) 620.
6. K. Prassides *et al.*, *Nature* **354** (1991) 462.
7. M.G. Mitch, S.J. Chase and J.S. Lannin, *Phys. Rev. Lett.* **68** (1992) 883.
8. W. Weber, *Phys. Rev. Letters* **58** (1987) 1371.
9. W. Weber and L.F. Mattheiss, *Phys. Rev.* B **37** (1988) 599.
10. H. Maeda *et al.*, *Japn. J. Appl. Phys.* **27** (1988) L209.
11. Z.Z. Sheng and A.M. Hermann, *Nature* **332** (1988) 55.
12. C.H. Eab and ImM. Tang, *Phys. Rev.* B **40** (1989) 4427.
13. J.L. Birman and J.P. Lu, *Phys. Rev.* B 39 (1989) 2238.
14. R. Kleiner and P. Muller, *Phys. Rev.* B **49** (1994) 1327; R. Kleiner *et al.*, ibid. 50 91994) 3942.
15. D.N. Basov *et al.*, *Phys. Rev.* B **50** (1994) 3511.
16. F. Zuo *et al.*, *Phys. Rev. Lett.* **72** (1994) 1746.
17. Z. Tesanvic, *Phys. Rev.* B **36** (1987) 2364.
18. Q. Huang *et al.*, *Nature* **347** (1990) 369.
19. W.L. McMillan, *Phys. Rev.* **167** (1968) 331.
20. D.R. Harshman and A.J. Mills, Jr., *Phys. Rev.* B **45** (1992) 10, 684.