A MICROSCOPIC THEORY FOR THE INTERLAYER ENHANCEMENT OF SUPERCONDUCTIVITY IN THE HTSC'S

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(Received October 20, 1995)

ABSTRACT

The effects of interlayer interaction through the particle-particle channel and the particle-hole channel on an electron-phonon (breathing mode) model of the high $T_c$ superconductor are studied. The strong coupling Eliashberg formalism is used in the study. It is shown that the interaction through the former channel can appreciably enhance the $T_c$ of the two CuO$_2$ layer superconductor (YBa$_2$Cu$_3$O$_7$) over that of the single layer superconductor ($\text{La}_{2-x}\text{(Sr,Ba)}_x\text{CuO}_4$). The calculations are done in the weak coupling limit (the $\lambda$\textsuperscript{00} model) of the Eliashberg formalism.

INTRODUCTION

The discovery of superconductivity in the doped fullerenes [1] has created much excitement in the field since pair condensation in the fullerene was so unexpected. The highest $T_c$ achieved, that of Cs$_x$Rb$_y$C$_{80}$, [2] is comparable to the $T_c$ of the first cuprate high $T_c$ superconductor discovered, La$_{2-x}$Ba$_x$CuO$_4$ [3] (33 K versus 35 K). While the isotope effect in La$_{2-x}$Ba$_x$CuO$_4$ is greatly reduced, [4] it is quite large in the fullerene superconductor, Rb$_3$C$_{80}$. Ebbesen et al., [5] obtained an isotope shift exponent of $1.4 \pm 0.5$. This value and somewhat smaller values obtained by others point to the electron-phonon interaction as being the dominant mechanism for superconductivity in the doped fullerene. In fact, it has been established that the active phonons are the higher frequency intramolecular H$_{g}$ modes. [6, 7]

In one of the earlier theories [8, 9] for superconductivity in the cuprate high $T_c$ superconductors (HTSC), it was proposed that the pairing was via the high frequency oxygen breathing mode vibration. Weber [8] studied the el-ph interactions in La$_{2-x}$(Ba, Sr)$_x$CuO$_4$ and concluded that the high $T_c$'s of these superconductors were not caused by the unusually strong el-ph coupling, but were caused by the fact that the $2F(\ )$, the spectral density function, extends to much higher frequencies than in the case for typical transition-metal superconductors. Weber and Mattheiss [9] studied the el-ph interaction in the YBa$_2$Cu$_3$O$_7$ and found that the el-ph coupling would not be strong enough to account for a $T_c$ of 90 K. They closed their paper with the remark, "An additional or alternative mechanism for electronic pairing is required."
Insertion of additional CuO$_2$ layers into the bismuth and thallium superconductors increased the T$_c$'s of these superconductors. [10, 11] Using the Ginzburg Landau approach, two groups showed that these increases in the T$_c$'s could be accounted for by an interlayer coupling of the order parameters belonging to adjacent layers [12, 13]. Eab and Tang [12] took the interlayer coupling to be due to a nearest layer bilinear Josephson coupling, while Birman and Lu [13] took the coupling to be due to some type of nearest layer spin-spin interaction. Recently, clear evidences for Josephson tunneling between CuO$_2$ layers has been seen in the single crystal superconductors, Bi$_2$Sr$_2$CaCu$_2$O$_{8}$, (Pb$_y$Bi$_{1-y}$)$_2$Sr$_2$CaCu$_2$O$_{8}$ and Tl$_2$Ba$_2$Ca$_2$Cu$_3$O$_{10}$ [14]. The magnetic field dependence of the c-axis critical currents in the bismuth superconductors pointed to Josephson pair tunneling between superconducting layers (formed by the double CuO$_2$ layers) separated by 15 A thick insulating layers (formed by the Bi$_2$O$_3$ and SrO layers). Microwave emission measurements indicate that these three HTSC's could be considered to be stacks of intrinsic SIS Josephson junctions. Evidences for Josephson tunneling in other single crystal HTSC's, YBa$_2$Cu$_4$O$_8$ and Nd$_{1.85}$Ce$_{0.15}$Cu$_2$O$_4$ have been seen by Basov et al., [15] and by Zuo et al., [16] respectively.

In one of the first microscopic studies on these interactions in the high T$_c$ superconductors, Tesanovic [17] showed that the interlayer interaction in the particle-particle channel could enhance the T$_c$. The purpose of this paper is to examine the effects of the interlayer coupling on the HTSC within the framework of the strong coupling approach. It is assumed that the primary mechanism for superconductivity is the el-ph mechanism with the active phonon being the high frequency oxygen breathing mode.

ELIASHBERG EQUATIONS

Since most evidences point to the cuprate high T$_c$ superconductors being strong coupling superconductor, the Eliashberg formalism should be used to study these superconductors. To begin, we first note that the coupling can be through either the particle-hole channel or the particle-particle channel.[17] The following perturbations

\[ t_1 \sum_{k, k'; \sigma} C^*_{k, \sigma, i} C_{-k, -\sigma, j} C_{k', -\sigma, j} C_{-k', \sigma, i} \]  \hspace{1cm} (1)

and

\[ t_2 \sum_{k, k'; \sigma} C^*_{k, \sigma, i} C_{-k, -\sigma, i} C_{k', -\sigma, j} C_{-k', \sigma, j} \]  \hspace{1cm} (2)

represent the interactions through the two channels, respectively. In the above, $t_1$ and $t_2$ are the two interlayer coupling strengths; k and k', the momentums; $\sigma$, spin labels and i and j label adjacent layers. These interactions are assumed not to have originated from the direct hopping of an electron from one layer to another. Instead, it was suggested in ref. 17, that the interaction in the particle-hole channel arises from either from plasmon,
exciton or phonon assisted transitions and that the interaction in the particle-particle channel generates "Josephson-like tunneling" coupling of order parameters belonging to adjacent layers.

The screened interaction appearing in Fig. 1a, is generated by the particle-hole channel interaction, while the screened interaction appearing in Fig. 1b, is generated by the particle-particle channel interaction. The bubbles in the screened interactions are the two particle propagators with different vertices. The diagrams in Figs. 1a and 1b represent the diagonal and off diagonal parts of the self energy corrections due to the interlayer coupling. These should be included with the corrections arising from the el-ph interaction and from the Coulomb interaction. When done, the following Eliashberg equations are obtained

\[
\{Z(i\omega_n) - 1\} \omega_n = \pi k_B T \sum_m \lambda_{(n-m)} \frac{\omega_m}{\omega_m^2 + |\Delta_m|^2}^{1/2} \\
+ \pi k_B T \sum_{m} \frac{\omega_m}{\omega_m^2 + |\Delta_m|^2}^{1/2}
\]

and

\[
Z(i\omega_n) \Delta_n = \pi k_B T \sum_{m} (\lambda_{(n-m)} - \mu^* \theta (\omega_{\text{cutoff}} - |\omega_m|) \frac{\Delta_m}{\omega_m^2 + |\Delta_m|^2}^{1/2} \\
+ k_B T \sum_{m} \frac{\Delta_m}{\omega_m^2 + |\Delta_m|^2}^{1/2}
\]

where \(\omega_m = i \pi k_B T (2m + 1)\). \(\mu^*\) is the Coulomb pseudo potential defined by

\[
\mu^* = \mu / \{ 1 + \mu \ln (\omega_{\text{cutoff}}/\pi T_c) \}
\]

with the cutoff frequency being the breathing mode frequency for the case of HTSC. The el-ph pairing parameter is given by

\[
\lambda_{(n-m)} = 2 \int \frac{\Omega d\Omega \alpha^2 F(\Omega)}{\Omega^2 + (\omega_m - \omega_n)^2}
\]

Fig. 1. Self Energy Correction. 1a. Self energy correction due to the interlayer coupling in the particle-hole channel. 1b. Correction due to the interlayer coupling in the particle-particle channel.
Fig. 2. Vertex Correction. 1a. The Bethe-Salpeter equation for the vertex appearing in the bubble diagram appearing in the self energy correction defined by Fig. 1a. 1b. The Bethe-Salpeter equation for the vertex appearing in the bubble diagram occurring in Fig. 1b. It is assumed that the main correction to the vertex is the exchange of a phonon.
with $\alpha^2 F(\Omega)$ being the spectral density function for the superconductor. These functions for $\text{La}_{2-x}\text{Ba}_x\text{Cu}_4\text{O}_7$ and $\text{YBa}_2\text{Cu}_3\text{O}_7$ have been calculated theoretically by Weber [8] and Weber and Matheiss, [9] respectively, while the spectral density function for $\text{Nd}_{2-x}\text{Ce}_x\text{Cu}_4\text{O}_7$ has been determined experimentally by Huang et al. [18]. $T_1$ and $T_2$ are the interlayer coupling energies defined as

$$T_1 = \pi k_B T \sum_m \int d^3p \ t_1^2 G(p, i\omega_m) G(-p, -i\omega_m) \gamma_1(i\omega_m)$$

and

$$T_2 = \pi k_B T \sum_m \int d^3p \ t_2^2 F(p, i\omega_m) F(-p, -i\omega_m) \gamma_2(i\omega_m)$$

where $\gamma_1$ and $\gamma_2$ are the vertices defined by Figs. 2a and 2b, respectively. $G(p, i\omega_m)$ and $F(p, i\omega_m)$ are the single particle propagators.

**$T_C$ ENHANCEMENT**

Combining the Eliashberg equations together, we get

$$\Delta_n \{ 1 + \pi k_B T \sum_{m = -\infty}^{\infty} \left[ \lambda(n-m) + T_1 \right] \} \frac{\Delta_m}{[\omega_m^2 + 1 \Delta_m^2]^{1/2}} = \pi k_B T \sum_{m = -\infty}^{\infty} \left[ \lambda(n-m) - \mu^* \theta (\omega_{\text{cutoff}} - |\omega_m|) + T_2 \right] \frac{\Delta_m}{[\omega_m^2 + 1 \Delta_m^2]^{1/2}}$$

The simplest solution is achieved in the square well $\lambda^{\Theta}$ model (the weak coupling limit) where the el-ph pairing parameter $\lambda(n-m)$ is approximated by

$$\lambda(n-m) = \theta (\omega_{\text{cutoff}} - |\omega_n|) \lambda \theta (\omega_{\text{cutoff}} - |\omega_m|)$$

with $\lambda = \lambda(0)$. With this assumption, it follows that the order parameter $\Delta_n$ is independent of $\omega_n$, i.e., the order parameter is a constant. We will also assume that $T_1$ and $T_2$ are independent of $\omega_n$. Working in the weak coupling limit of Eliashberg approach, we get

$$T_C \sim \omega_{\text{cutoff}} \exp \left\{ (1 + \lambda + T_1) / (\lambda - \mu^* + T_2) \right\}$$

where $\omega_{\text{cutoff}}$ is the highest oxygen breathing mode frequency and $\lambda$ is the el-ph coupling strength for the HTSC.
As we have pointed out, Weber [8] has calculated the spectral density function $\alpha^2(\Omega)$ for $\text{La}_{2-x}(\text{Sr, Ba})_x\text{CuO}_4$. He found that the function extends up to frequencies of 80 meV or so. The higher frequencies arise from the O(1) and O(2) stretching modes. Weber calculated the el-ph coupling strength for the La superconductor and found it to be in the range 2 to 2.5, similar to the values of for the A15 compounds, which have low $T_c$'s. Using a value of 0.13 for the Coulomb pseudo potential $\mu^*$, a $T_c$ of between 30 and 40 K was predicted using the standard $T_c$ formula [19].

$$T_c = \hbar \omega_{\text{cutoff}} \exp \left\{ (1 + \lambda) / (\lambda - \mu^*) \right\} \quad (11)$$

Performing a similar calculation, Weber and Mattheiss [9] obtained the spectral density function for $\text{YBa}_2\text{Cu}_3\text{O}_7$. They found the el-ph coupling strength for $\text{YBa}_2\text{Cu}_3\text{O}_7$ superconductor to be between 1.3 and 2.3. Using the same value the Coulomb pseudo potential $\mu^*$, a $T_c$ of between 19 and 30 K was predicted for $\text{YBa}_2\text{Cu}_3\text{O}_7$. The lower $T_c$ is also due in part to the fact that the spectral density function extends up to 70 meV only. Weber and Mattheiss point out that increasing the value of $\lambda$ could not lead to the needed increase in $T_c$ since the presence of $\lambda$ in both the numerator and denominator of the exponential factor in eqn. (11), causes a saturation of the $T_c$ values. They concluded that the jump in the $T_c$ of $\text{YBa}_2\text{Cu}_3\text{O}_7$ from 19-30 K to 90 K would have to be due to the presence of an additional mechanism in $\text{YBa}_2\text{Cu}_3\text{O}_7$ which is not present in $\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$.

The expression for the transition temperature given by eqn.(11) does not contain any parameters which are explicitly connected to the main difference between the two HTSC's, i.e., the presence of a double CuO$_2$ layer in $\text{YBa}_2\text{Cu}_3\text{O}_7$ as oppose to only a single CuO$_2$ layer in $\text{La}_{2-x}(\text{Sr, Ba})_x\text{CuO}_4$. Equation 10, however, does. To bring out the difference between eqns. (10) and (11), we combine the two to obtain

$$T_c = T_c^* \exp \left\{ (1 + \lambda) / (\lambda - \mu^*) - (1 + \lambda + T_1) / (\lambda - \mu^* + T_2) \right\} \quad (12)$$

where $T_c^*$ is transition temperature in the absence of the interlayer interactions, i.e., the transition temperature given by eqn. (11). Figures 3a and 3b show the plots of $T_c$ versus the interlayer coupling energies $T_2$ for different values of $T_1$. The value of the Coulomb pseudo potential is set at 0.13. In Fig. 3a, we plot the dependence of $T_c$ on $T_2$ when the el-ph coupling strength is set at 1.3 and 2.3, the lower and upper values of the el-ph coupling strength given by Weber and Mattheiss [9] for $\text{YBa}_2\text{Cu}_3\text{O}_7$ and $T_1$ is set to zero. As we see, $T_2$, the interlayer energy arising from the coupling in the particle-particle channel, can enhance the $T_c$ appreciably, the effect being more evident for smaller values of $\lambda$. In Figure 3b, we plot the dependence of $T_c$ for different values of $T_1$, the interlayer energy arising from the coupling in the particle-hole channel. The value of el-ph coupling strength used 1.3. As we see, $T_1$ causes a lowering of the transition temperatures.
Fig. 3. Dependence of T$_c$. On the interlayer interaction T$_1$. and T$_2$. 3a. Dependence of T$_c$ on the interlayer interaction via the particle-particle channel, T$_2$. The two values of $\lambda$ are the upper and lower values of the electron-phonon strengths given by Weber and Mattheiss. [9] 3b. Effect of interlayer interaction in the particle-hole channel, T$_1$, on T$_c$. 
Egn. 12 and the dependence of the enhancement of $T_c$ on the value of interlayer interaction strength $T_2$ exhibited by Fig. 3a may also provide a possible framework for understanding the differences in the values of the $T_c$s of all two copper layer HTSCs containing the CuO$_2$/Ca/CuO$_2$ layer. The strength of the Josephson coupling across a barrier of thickness $d$ goes as $d^{-2}$, meaning that as the distance between the CuO$_2$ layers decreases, the Josephson coupling strength should increase leading to a higher $T_c$. Harshman and Mills [20] considered all novel superconductors in their attempt to find correlations between the various parameters of the superconductors. They found the $T_c$s of most of the members of the above set of HTSC's correlates with $d^{-1}$ (See Fig. 12b of ref. 20). The dependence of $T_c$ on the distance between the CuO$_2$ layers implied by eqn. (12) would be much more complicated than a simple $d^{-1}$ or $d^{-2}$ dependence since other parameters such as the breathing mode frequency and the el-ph coupling strengths may also depend on $d$. Nevertheless, the increased enhancement of $T_c$ seen in Figs. 3a and 3b is consistent with the correlation observed in ref. 20.

ACKNOWLEDGEMENT

One of the authors (IMT) would like to thank the The Thailand Research Fund for financial support.

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