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FINITE ELEMENT ANALYSIS OF OPTICAL DIRECTIONAL COUPLERS

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ABSTRACT

Results are presented on a study of the important parameters of synchronous and nonsynchronous, weakly and strongly coupled optical directional couplers using the finite element method. Accurate propagation constants and field profiles have been obtained for the modes of the isolated guides and supermodes of the coupled system. The power transfer efficiency and cross-talk are calculated from the individual guide modes using improved coupled mode approximations and from the supermodes using the least squares boundary residual method.

I. INTRODUCTION

The investigation of coupling between optical waveguides has been a subject of considerable interest in the design of directional couplers [1], modulators and switches [2], wavelength filters [3], and large laser arrays [4]. The accurate calculation of coupling parameters is of considerable interest, to study the loss of synchronism in electro-optic modulators and switches or in optical filters in the use of nonidentical sections to reduce bandwidths.

In most of the practical directional coupler-based devices, the individual waveguides use two-dimensional confinement and can be of arbitrary cross-section with anisotropic, nonlinear, lossy or active materials. The finite element method [5] has already been established as one of the most powerful methods available to characterize a wide range of practical waveguides. This method has been used to find the coupling length accurately between two identical [6] and nonidentical [7] waveguides with two-dimensional confinement.

In this paper, the power coupling efficiency between two optical waveguides is presented for the first time using the finite element method along with the improved coupled mode approaches [8,9,10] and the least square boundary residual method [11].

II. THE FINITE ELEMENT METHOD

In the finite element method (FEM), the waveguiding region is subdivided into a patchwork of a finite number of subregions called elements. Each element can have different sizes and shapes, so using many such elements a complex waveguiding cross-section can be accurately represented and as each element can have a different refractive index, waveguides with arbitrary refractive index profiles can also be accurately modelled. Each element can also have a different loss or gain factor, different anisotropy or different nonlinearity so a wide range of practical waveguides can be considered. The analysis can be carried out by using the accurate vector H-field [5] formulation which is "exact-in-the-limit" or by using the approximate TE or TM scalar formulation which provides a faster solution.

The finite element method can provide accurate modal field profiles and propagation constants for all the modes of each individual guide or for all the supermodes for a coupled structures consisting of two or many waveguides. The modes in two isolated waveguides "*a*" and "*b*" may be considered as

$$\mathrm{E_a}(x,y,z) = E_a\,(x,y)\,\exp\,(j\beta_a z) \qquad (1a)$$
$$\mathrm{E_b}(x,y,z) = E_b\,(x,y)\,\exp\,(j\beta_b z) \qquad (1b)$$

respectively, where $E_a(x,y)$ and $E_b(x,y)$ describe the transverse (x,y) dependence of the modal eigenvectors and β_a and β_b are their propagation constants for isolated guides "*a*" and "*b*" respectively. Similarly the first two even and odd supermodes in the coupled structure can be considered as

$$\mathrm{E_e}(x,y,z) = E_e\,(x,y)\,\exp\,(j\beta_e z) \qquad (2a)$$
$$\mathrm{E_o}(x,y,z) = E_o\,(x,y)\,\exp\,(j\beta_o z) \qquad (2b)$$

where E_e (x,y) and E_o (x,y) describe the transverse dependence of the field profiles and β_e and β_o are their propagation constants for the even and odd supermodes of the coupled system. In the finite element solution, the entire coupled system is considered, so the resulting eigenvectors E_e (x,y) and E_o (x,y) are always orthogonal to each other, even when the guides are not identical and are strongly coupled. The supermode eigenvalues β_e and β_o are accurately calculated using the finite element method to obtain the coupling length accurately from the difference of β_e and β_o.

The main emphasis of this paper is the calculation of the power transfer efficiency between two coupled optical waveguides. It is possible to find the power transfer between two guides starting from the individual modes of the isolated guides or from the supermodes of the complete coupled structure. In this paper both such approaches are used, after obtaining accurate eigenvalues and eigenvectors of the individual guides and coupled structures.

III. THE COUPLED MODE APPROACH :

Once the transverse dependences of E_a (x,y) and E_b (x,y) are known along with their propagation constants, then the power transfer efficiency between the guides can be calculated by using various coupled mode approaches.

Traditional coupled mode theory [12] recently has been improved by Hardy and Streifer [8], by Marcatili [9] for nonidentical waveguides and by Chuang [10] for strongly coupled waveguides.

The mode overlap coefficients, C_{ij}, (i,j=a,b) which measure the proximity of the guides are given by [8]

$$C_{ab} = 2 \iint_{-\infty}^{+\infty} E_t^{(b)} \times H_t^{(a)} \cdot \hat{z}\, dx\, dy \tag{3a}$$

$$C_{ba} = 2 \iint_{-\infty}^{+\infty} E_t^{(a)} \times H_t^{(b)} \cdot \hat{z}\, dx\, dy \tag{3b}$$

Hardy and Streifer [8] and Chuang [10] calculate the overlap integral from the Poynting vector of **E x H**, whereas Marcatili [9] calls a similar parameter the butt coupling and calculates the mode overlap from the overlap integral of the electric fields which yields only a slight difference between the definitions, as β_a may not be the same as β_b for nonidentical guides.

The coupling coefficient from guide "*b*" to guide "*a*" is identified by K_{ab} which is given by [8].

$$K_{ab} = \frac{\{\tilde{K}_{ab} + C_{ab}[\beta^{(a)} - \beta^{(b)} - \tilde{K}_{bb}]\}}{(1 - C_{ab}C_{ba})} \tag{4a}$$

$$K_{ba} = \frac{\{\tilde{K}_{ba} + C_{ba}[\beta^{(b)} - \beta^{(a)} - \tilde{K}_{aa}]\}}{(1 - C_{ab}C_{ba})} \tag{4b}$$

where

$$\tilde{K}_{ab} = \omega \iint_{-\infty}^{+\infty} \Delta\varepsilon^{(a)} \left[E_t^{(a)} \cdot E_t^{(b)} - \frac{\varepsilon^{(b)}}{\varepsilon_0 n^2} E_z^{(a)} E_z^{(b)} \right] dx\, dy \tag{5a}$$

$$\tilde{K}_{ba} = \omega \iint_{-\infty}^{+\infty} \Delta\varepsilon^{(b)} \left[E_t^{(b)} \cdot E_t^{(a)} - \frac{\varepsilon^{(a)}}{\varepsilon_0 n^2} E_z^{(b)} E_z^{(a)} \right] dx\, dy \tag{5b}$$

These represent an improved definition of the coupling coefficient [8] when compared to $\tilde{K}_{ab}$ and $\tilde{K}_{ba}$ used in the conventional coupled mode theorem [12]. Marcatili [9] uses the notations K_a and K_b for parameters similar to K_{ab} and K_{ba} respectively. For coupling between two identical waveguides, K_{ab}, the coupling coefficient from guide *b* to guide a is identical to K_{ba}, which is the coupling coefficient from guide *a* to guide *b*.

Once the mode overlap coefficients, C_{ab} and C_{ba} and the coupling per unit length K_{ab} and K_{ba} are known, then the power transfer efficiency from guide *a* to guide *b* can be calculated by using equations (25) and (26) in the work of Chuang [10], equations (14) and

(15) in the work of Hardy and Streifer [8] and the formulae for P_a and P_b from the work of Marcatili [9].

IV. LEAST SQUARES BOUNDARY RESIDUAL METHOD

Since the FEM can provide accurate solutions for the supermodes of the coupled system, an alternative to the coupled mode approach as described in section III, the Least Squares Boundary Residual (LSBR) method has been applied in this paper for the first time to directional coupler problems. This procedure is used to find the power carried by the even and odd supermodes for a given incident power in guide "*a*" or "*b*". Here it is assumed a single isolated waveguide section, section I, is butt coupled to the directional coupler section, section II, as shown in Fig. 1. The main objective is to calculate the amplitudes of the even and odd modes b_1 b_2 respectively in section II. This approach is better than the use of traditional overlap integral methods as many modes can be considered to satisfy the field continuity at the discontinuity junction plane. This approach is also better than the point matching methods because the error integral is evaluated over the discontinuity interface, rather than just field matching at some specific points.

More detailed discussion on the Least Squares Boundary method can be found in our earlier work [11]. Briefly, the LSBR method looks for a stationary solution to satisfy the continuity conditions of both the tangential fields namely, E_t and H_t, in a least squares sense by minimizing the error function, J where

$$J = \int \left|E_t^I - E_t^{II}\right|^2 + \alpha Z_o^2 \left|H_t^I - H_t^{II}\right|^2 d\Omega \qquad (6)$$

where E_t^I, E_t^{II} and H_t^I, H_t^{II} are the transverse electric and magnetic fields in sections I and II respectively. Z_0 is the free-space wave impedance and α is a dimensionless weighting factor. It can be shown that the minimum criterion of equation (6) reduces to the following linear equation [11] :

$$Cx = v \qquad (7)$$

where [C] is square matrix generated from the eigenvectors and {v} is an array due to the incident mode.

The solution of this equation gives in {x} the required approximate coefficients of a_i and b_i. These constitute one column of the scattering matrix, corresponding to the chosen incident mode.

The eigenvalues and eigenvectors used are first generated by our vector FEM program. The eigenvectors are given by the nodal values of the three components of the vector **H** field for each mode. From these nodal **H** fields, the vector **E** field over each element can be

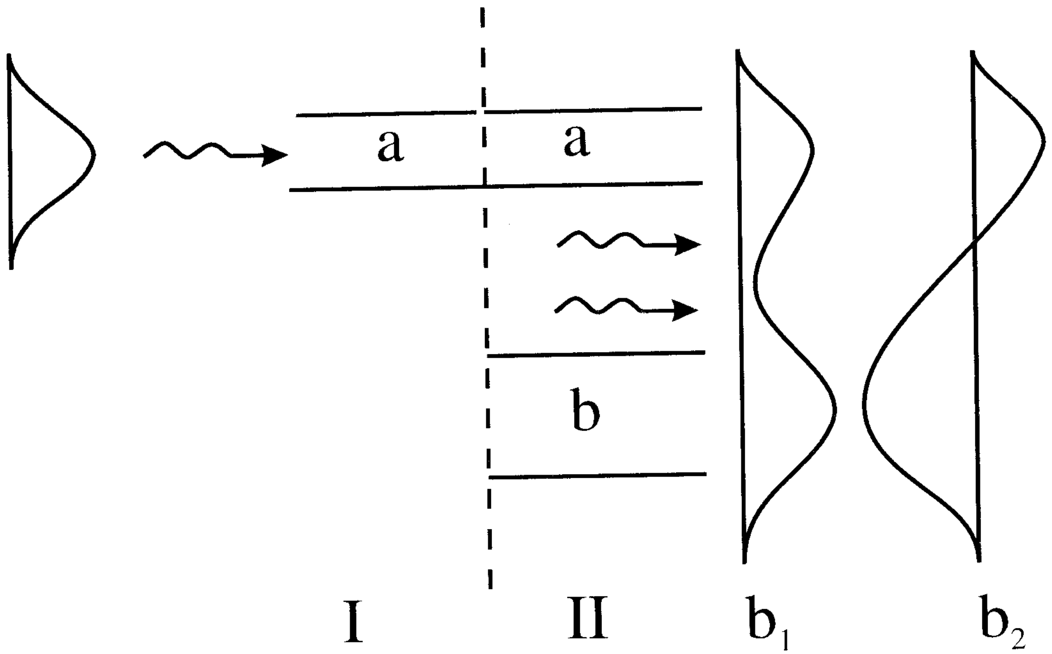

Fig.1. Butt coupling of an isolated guide to the directional coupler section.

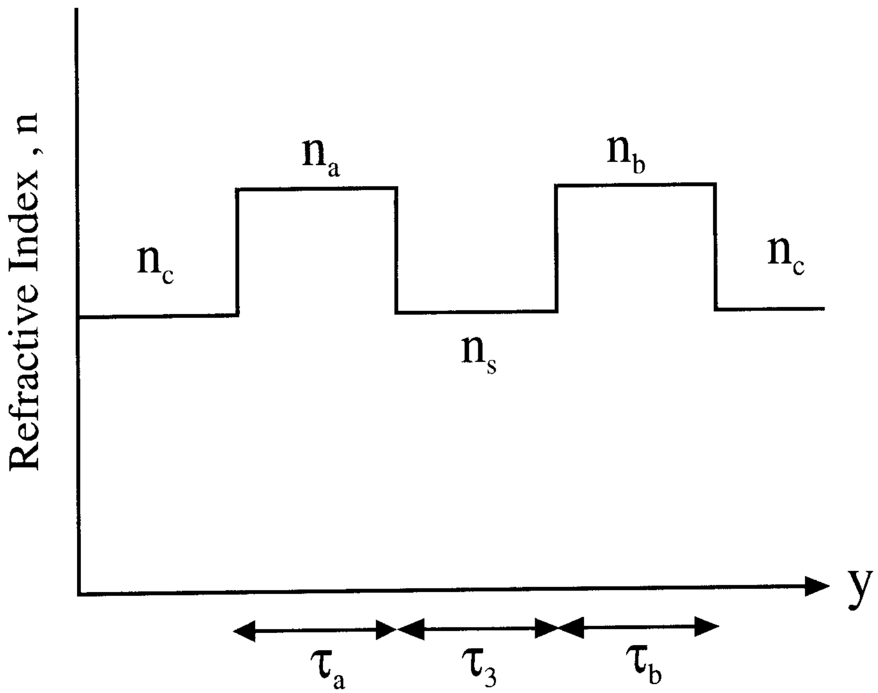

Fig.2. Schematic diagram of two parallel coupled optical waveguides.

calculated by applying Maxwell's equation. Many modal eigenvalues and eigenvectors for both sides of the discontinuity plane are used as the input to the LSBR program. The LSBR program calculates the error function J and minimizes the error criterion (6) with respect to each value of a_i and b_i for any given incidence, by solving a homogeneous linear equation (7). Solving this equation will give the unknown column vector {x} consisting of the unknown reflected and transmitted coefficients of all the modes considered in the analysis. The singular value decomposition algorithm has been used to solve the linear equation, (7). To improve the numerical efficiency, the FE nodal points in section I are matched with the nodal points of section II across the discontinuity interface. In this case there is no need to generate the nodal **E** fields, as the electric field part of the integral J in equation (6) can be calculated directly from the nodal **H** field values.

V. RESULTS

The finite element method is one of the most powerful methods available to obtain the propagation constants and modal profiles for a wide range of optical waveguides [5] including linear, nonlinear, anisotropic, and lossy optical waveguides of regular or irregular cross-section. However, here the results are restricted to the TE modes in planar waveguides, only to enable us to compare our results with other work, but it should be stressed that this numerical procedure is equally valid for hybrid modes in coupled waveguides with two dimensional confinement.

At first, the accuracy of the finite element method as a means to obtain propagation constants for the individual modes and supermodes is illustrated. In the first example, the structure analysed by Hardy and Streifer [8] is considered, which is shown in Fig. 2. Two slab waveguides "*a*" and "*b*" with film thicknesses τ_a and τ_b are separated by τ_3 (μm dimensions). The refractive indices for the guides *a* and *b*, the separation and the cladding region are n_a, n_b, n_s and n_c respectively and the operating wavelength is 0.8 μm.

Table 1 shows the comparison of the finite element solutions with the analytical solutions for the individual TE mode of the isolated guide "*b*" and the even and odd TE supermodes of the coupled structure. In this example, two waveguides with identical

TABLE 1. The comparison of finite element solutions (FEM) with analytical solutions (AN) for β_b, β_e and β_o.

τ_b	β_b		β_e		β_o	
in μm	AN	FEM	AN	FEM	AN	FEM
0.10	26.97534	26.97534	27.20137	27.20137	26.93143	26.93143
0.12	27.06138	27.06138	27.20992	27.20992	27001436	27001436
0.15	27.18799	27.18798	27.24361	27.24360	27.11346	27.11346
0.18	27.30535	27.30543	27.32241	27.32241	27.15637	27.15636
0.20	27.37685	27.37685	27.38579	27.38578	27.16669	27.16668

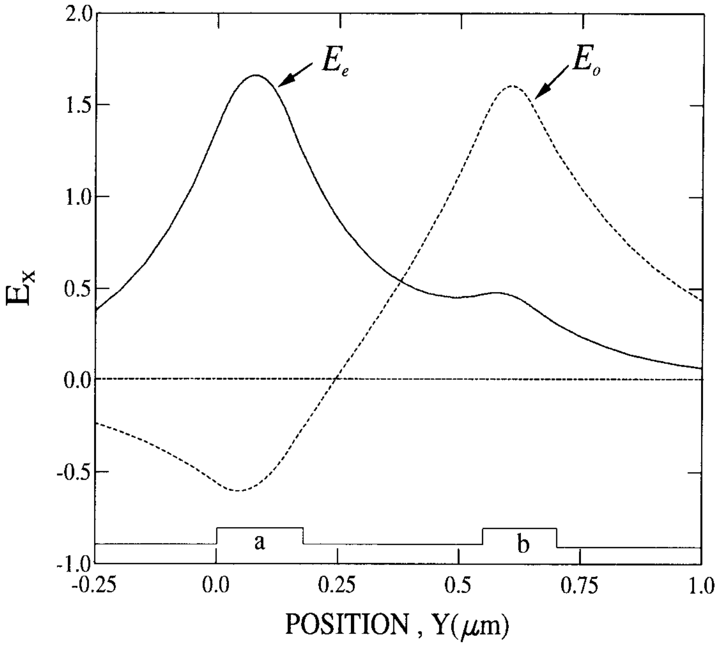

Fig.3. E_x field profiles for the even- and odd- like TE supermodes.

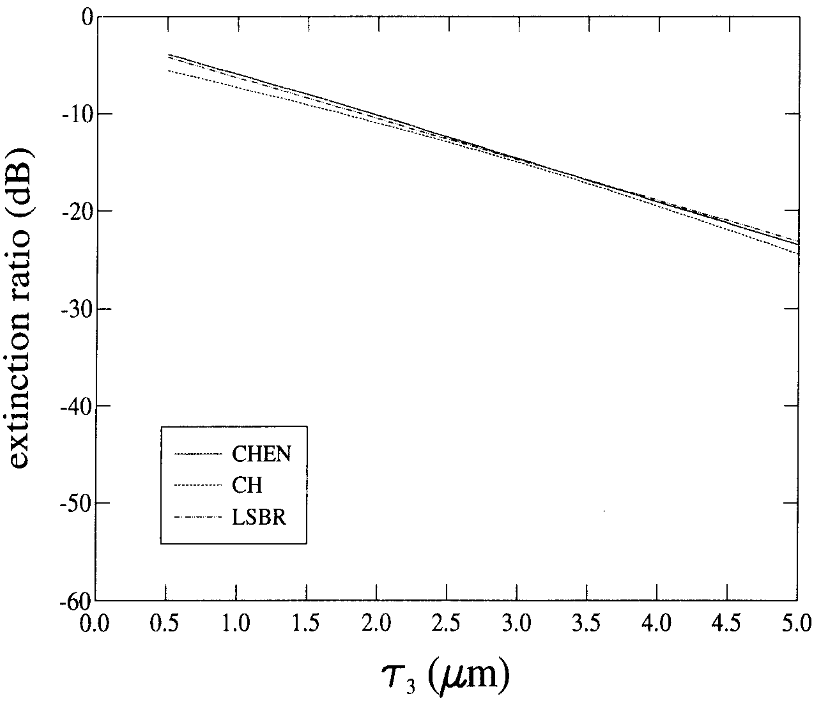

Fig.4. Cross-talk between the guides with the separation distance for coupled identical waveguides.

refractive indices, $n_a = n_b = 3.6$ and the cladding refractive index, $n_s = n_c = 3.4$ are considered. In this case τ_a and τ_3 are fixed at 0.15 μm and 0.4 μm respectively, but τ_b varies from 0.1 μm to 0.2μm. Here β_b is the propagation constant for the isolated guide b and β_e and β_o are propagation constants for the even and odd supermodes of the coupled guides. The analytical solutions (AN) are obtained by finding roots of the transcendental equation due to the field matching at the dielectric interfaces. The finite element (FEM) solutions are obtained by using 4000 mesh divisions. It takes about 10 seconds to find modal solutions on a SUN Sparcstation 2 for this mesh refinement. Table 1 shows the excellent agreement of the finite element results with the analytical results and if required, the accuracy can be further improved by using an even finer mesh.

Fig. 3 shows the Ex field profiles for the even- and odd-like TE supermodes for a nonsynchronous directional coupler when τ_a and τ_b are 0.15μm and 0.1 μm respectively, with the separation distance, τ_3 = 0.4 μm. It can be observed that the modal profiles are not symmetrical since the individual guides are not identical. The first supermode is the even-like mode with most of the power confined in the guide "a" whereas the second supermode is odd-like mode with most of the power confined in the guide "b". This is because the dominant mode in guide "a" has a higher propagation constant than the mode in guide "b" (as $\tau_a > \tau_b$) so the first supermode with higher propagation constant resembles more the mode E_a when the individual modes are not phase matched. The power intensity profile obtained from these field profiles match accurately with the exact profiles shown in Fig. 2 of the results of Hardy and Streifer [8].

To reduce the coupling length, the separation distance τ_3 can be reduced to increase the mode coupling and subsequently shorten the overall device length. However, this will introduce cross-talk due to incomplete power transfer between the guides, even when the guides may be identical. First a synchronous directional coupler problem [13] is considered to test our numerical procedures. In this second example, $n_a = n_b = 3.44$, $n_c = n_s = 3.436$, $\tau_a = \tau_b = 2.0$ μm, and the wavelength is considered to be 1.06 μm. Fig. 4 shows the cross-talk in dB with the separation distance, τ_3, using the coupled mode approach (CH) [10] and the Least Squares Boundary Residual method (LSBR) approach. In this example, results from both the approaches agree reasonably with the work of Chen [13].

To find the power transfer efficiency and cross-talk, using the coupled mode approaches, the overlap integrals and coupling coefficients need to be calculated. To illustrate the procedure and steps taken, some intermediate results are presented for the first example with coupled nonidentical waveguides, which is more complicated than in the second example, which is for the coupled identical waveguides. Fig. 5 shows the variation of K_{ab} and K_{ba} with the thickness of guide b, τ_b, when thickness of guide a and τ_3 are fixed at 0.15 μm and 0.4 μm respectively. In this figure, K_{ab} and K_{ba} are compared with the approximate coupling coefficients $\tilde{K}_{ba}$ and $\tilde{K}_{ab}$, which are calculated by using equation (5a and 5b) [8]. Next accurate values of K_{ab} and K_{ba} are calculated by using equation (4a and 4b) [8] by considering appropriate correction factors. K_{ab} reduces monotonically as τ_b increases, where as Kba increases. It can be observed that the coupling coefficients are identical when $\tau_a = \tau_b$, but when the guides are not identical, the coupling coefficients can differ widely. Our results agree well with those of Hardy and Streifer [8].

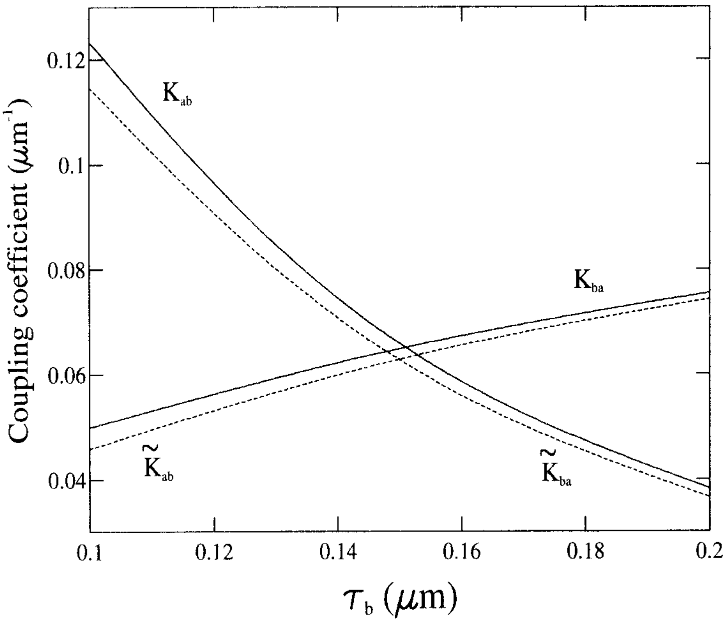

Fig.5. Variation of coupling coefficients with the second guide thickness, τ_b.

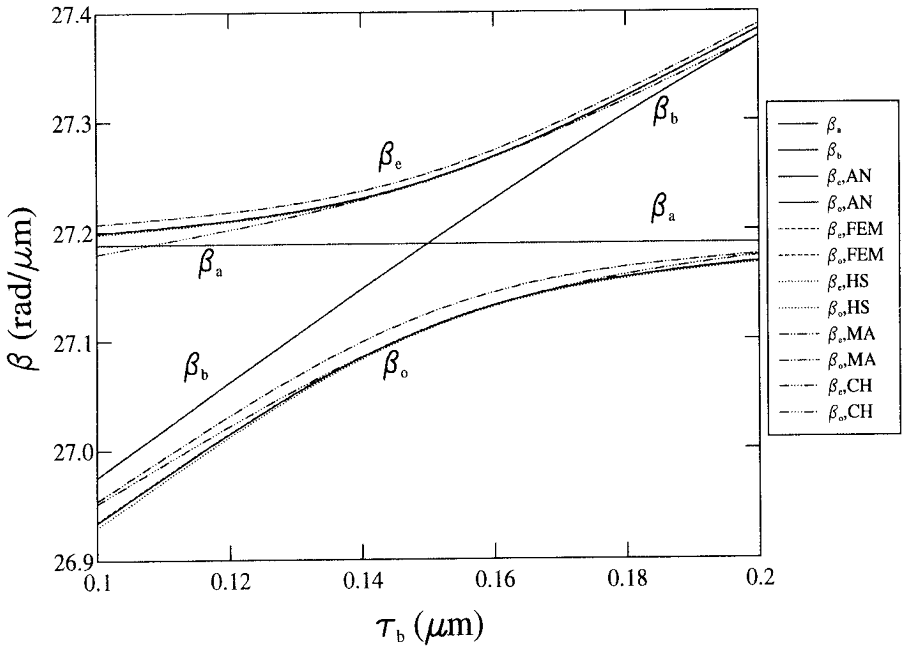

Fig.6. Variation of the calculated supermode propagation constants with the second guide thickness, τ_b.

The coupling length of the coupled waveguides can be accurately calculated from the propagation constant difference of the first two supermodes. In the finite element method the propagation constants of the two supermodes are obtained directly, whereas using the coupled mode theory, these can also be estimated from individual modes of the two isolated guides by using different coupled mode approaches.

Fig. 6 shows the variation of the propagation constants of the two supermodes with the second guide thickness, τ_b. The finite element results (FEM) and analytical results (AN) are identical and cannot be distinguished in this figure. Results using the Hardy and Streifer (HS) approach [8] agreed better with the analytical and the FEM results. Results using the Marcatili (MA) approach [9] are consistently higher value than the actual eigenvalues. The propagation constants of individual modes β_a and β_b are also shown. Results using the Chuang (CH) approach [10] are also satisfactory except when τ_b = 0.10 μm, when β_e is smaller than β_a, which cannot be correct.

Fig. 7 shows the coupling length variation with the second guide width, τ_b, when τ_a = 0.15 μm for different separation distances, τ_3. It has been shown in Table 1 that the analytical (AN) and the finite element results (FEM) agree extremely well even when the guides are strongly coupled and they cannot be distinguished one from another, whereas, calculation of the propagation constants of the supermodes by using the coupled mode approaches may be satisfactory but not very accurate, as shown in Fig. 6. The coupled mode (CH) approaches [10] agree well for the weakly coupled conditions but in general overestimate the coupling length when the guides are not synchronous, as shown for τ_3= 0.4 μm. It should also be noted that when $\tau_a = \tau_b$, the coupling length varies exponentially with the separation distance, τ_3, but when τ_a is not equal to τ_b, the coupling length depends mostly on the factor $|\beta_a - \beta_b|$.

The main emphasis of this paper has been the calculation of power transfer efficiency between the two optical waveguides. It has been mentioned earlier that the power transfer ratio can be obtained by starting from the individual modes of the isolated guides or from the supermodes of the complete coupled structure. Results are shown later in which both approaches are used, after obtaining accurate eigenvalues and eigenvectors of the individual guides and coupled guides.

First the amplitudes of the even and odd supermode field coefficients are calculated using the LSBR approach. Fig. 8 shows the variation of coefficients b_1 and b_2 with a second guide thickness, τ_b. When guide *a* is wider than guide *b*, ($\tau_b < \tau_a$), the amplitude of the odd-like supermode (b_2) is higher than that of the even-like supermode (b_1). Similarly, when guide *b* is wider than guide *a* ($\tau_b > \tau_a$) an even-like supermode carries more power than the odd-like supermode. It can be noted that when τ_b= 0.15 μm, although the guides are identical, but the two supermodes carry unequal power, where b_1 is equal to 0.786 and b_2 is equal to 0.617. This is due to the strong coupling between the isolated modes and their inequality is responsible for less than 100% power transfer between the guides. However, it has been tested by the authors (but not shown here) that for weakly coupled identical guides $b_1 \approx b_2 \approx \frac{1}{\sqrt{2}}$ and each supermode will carry half of the incident power in section I.

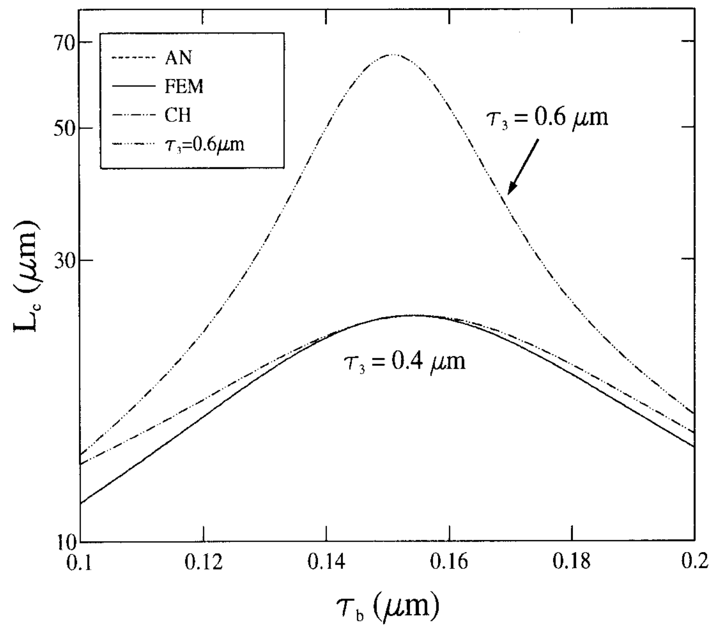

Fig.7. Variation of coupling length with the second guide thickness, τ_b for different separation between the guides, τ_3 (AN solution overlaps FEM solution).

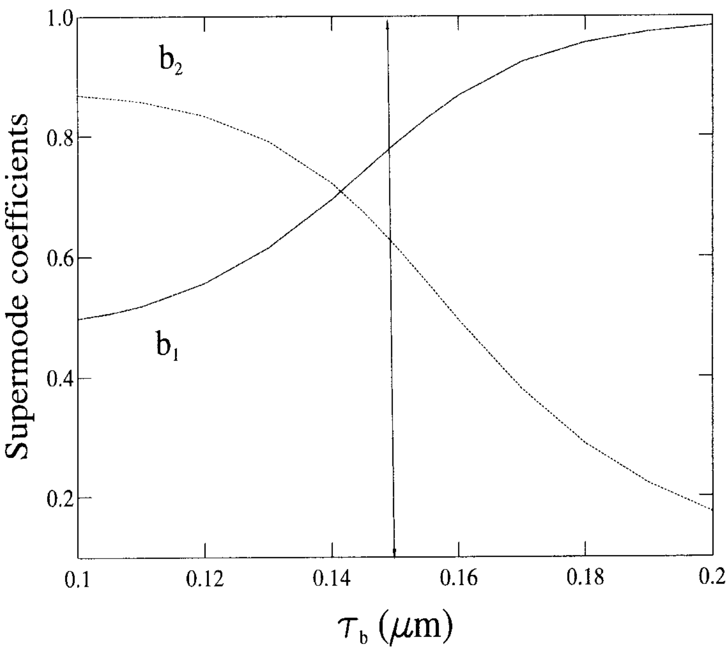

Fig.8. Variation of supermode amplitude coefficients, b_1 and b_2 with the second guide thickness, τ_b.

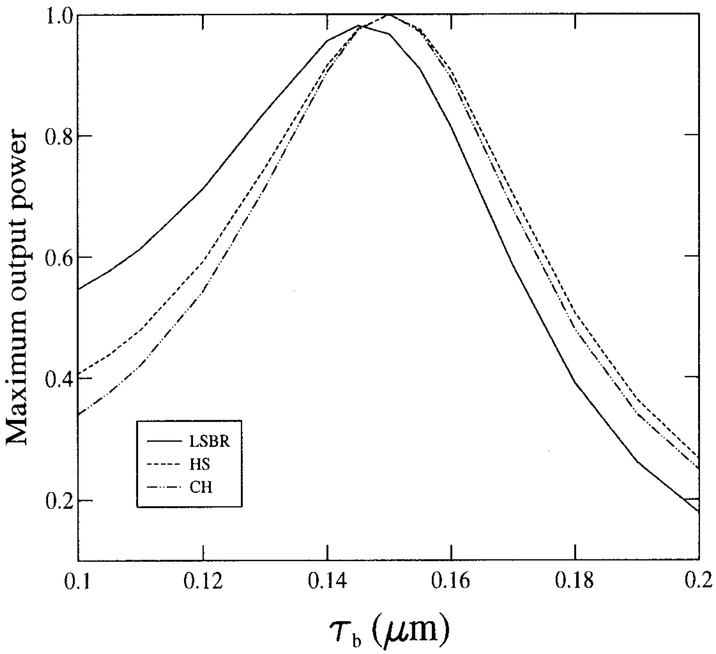

Fig.9. The maximum power transfer efficiency between the guides with the second guide thickness, τ_b.

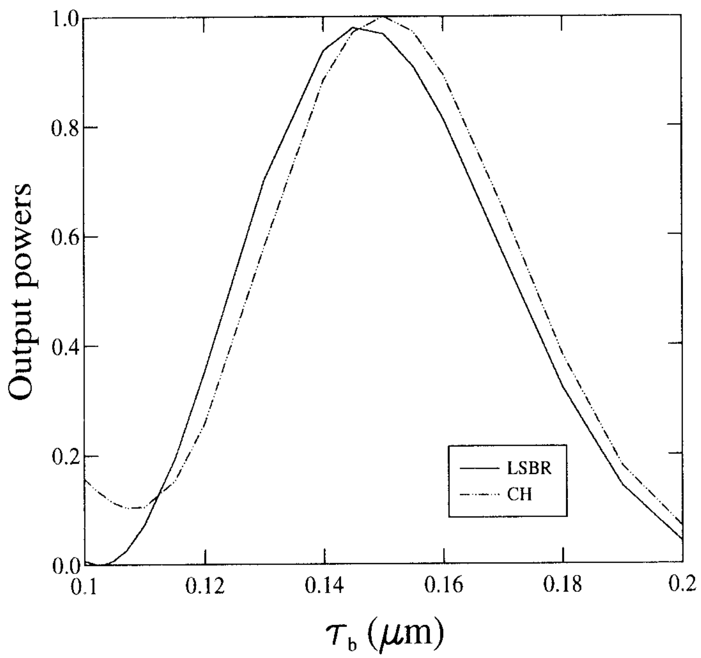

Fig.10. The power transfer efficiency between the guides with the second guide thickness, τ_b for a fixed length directional coupler devices.

Fig. 9 shows the variation of the maximum power transfer efficiency with τ_b using three different approaches. In this case the total length of the device is adjusted to be equal to the coupling length for different widths of guide b. Contrary to the coupled mode approaches, the LSBR result shows the maximum power transfer between the guides when τ_b is slightly smaller than 0.15 μm. However, it has been tested by the authors by using the LSBR approach (but not shown here) that maximum power transfer between the guides takes place when $\tau_b = \tau_a$, only if the guides are weakly coupled.

Fig. 10 shows the variation of the power transfer efficiency with τ_b using the coupled mode (CH) [10] and LSBR approaches. In this case, the total length of the device is fixed at L_c for $\tau_b = 0.15$ μm, where both the effect of the lack of phase synchronism and the change of the coupling length have been considered. This conditions simulates the loss of synchronism due to fabrication tolerances or due to external effects. Here, the power transfer efficiency is significantly lower than the maximum power transfer, as shown in Fig. 9. This reduced power transfer is due to the additional effect of coupling length mismatching as the value of L_c changes with τ_b whereas the device length is kept fixed.

VI. CONCLUSION

The finite element analysis provides a determination of the accurate coupling length for weakly or strongly coupled identical or nonidentical waveguides by using accurate vector formulations for parallel waveguides of arbitrary cross-section, dissimilar index and nonidentical shapes. The application of improved coupled mode theories along with the accurate eigenvectors and eigenvalues obtained by the finite element method provide the power transfer ratio between such coupled waveguides. Different coupled mode approaches have been considered to study some of the coupled guided structures. Here the results from the LSBR have also been presented, which agree well with the results from the coupled mode approaches. In this paper, the characterization of the TE mode coupling in planar structures are presented only to make a comparison with published results using different procedures. However, these approaches presented are valid for hybrid modes in optical waveguides with two-dimensional confinement. The calculation of important device parameters, such as power transfer, cross-talk, and filter bandwidth, for practical optical waveguides, will be very useful to optimize the design of modern directional coupler-based photonic devices.

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