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A STOCHASTIC MODEL FOR MAXIMUM DEPTHS OF DAILY RAINFALL

HUYNH NGOC PHIEN

Asian Institute of Technology, Bangkok, Thailand

PRATHET SUTABUTR

National Energy Administration of Thailand, Bangkok, Thailand

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Abstract

A mathematical model is proposed for the analysis of maximum depths of daily rainfall in the northeastern part of Thailand, using a Poisson distribution for annual exceedances and a shifted exponential distribution for the magnitudes of hazard events. These assumptions are justified by suitable statistical tests. An important relationship between the return periods of the annual maxima series and the partial-duration series is established, allowing investigators to relate the analysis based upon one series to that based upon the other. With this model, the Gumbel distribution is shown to be the distribution of annual largest exceedances when the average number of annual exceedances is equal to 1.

Introduction

The distribution of the maximum depths of daily rainfall is very important to the irrigation and drainage of agricultural lands. A sound understanding of this distribution is essential for the efficient use of available water and also for the design of drainage systems.

The analysis of the maximum depths of daily rainfall can be carried out using the annual maxima series and/or the partial duration series. The annual series consists of the maximum values in different years, one value per year. The use of this series is very convenient because of the fact that annual maximum values are readily available in most hydrological yearbooks. However, since it takes one value, and only one, each year from the record concerned, the second, third, etc... highest values in a particular year are disregarded even though they are higher than the annual maximum values in some other years. This disadvantage can be remedied by the use of the partial-duration series which comprises all values of the record exceeding some base value.

For the northeastern part of Thailand (hereafter referred to as the Northeast) where drought conditions last for several months and severe floods occur during periods of heavy rainfall in August and September[1,2], the analysis of the maximum depths of rainfall is even more important because the resulting information may be useful to the design of reservoir storage and to flood forecasting. However, only a few studies on these extreme values have been carried out, and the annual maxima and partial-duration series have been seprately dealt with. In this study, an attempt is made to develop a mathematical model for the analysis of these two series. Finally several rainfall records in the Northeast are used to show the applicability of the developed model.

The Model

Let X be the random variable representing the depth of daily rainfall and x_o be a particular value of X. In the following, x_o is treated as a *base value* so that any possible value x of X must be compared with x_o. When $x < x_o$, this value is automatically disregarded. When $x > x_o$, one has an *exceedance*, and X is then an exceeding event. In terms of damages, an exceeding event is normally called a *hazard event*. Only the exceeding events are considered.

For the analysis of extreme values such as the maximum depths of daily rainfall, an annual basis is commonly adopted. The proposed mathematical model then consists of the following two assumptions:

(i) The number N of exceedances in each year follows a Poisson distribution:

$$P\,[N = n] = e^{-\lambda}\lambda^{n}/n!\ ,\ n = 0,\ 1,\ 2.\ ... \qquad (1)$$

in which λ is a constant, called the *exceedance rate.*

(ii) The magnitudes of the hazard events are independent and identically distribution with a distribution function F(x):

$$P[X \leq x] = F(x),\ x > x_o \qquad (2)$$

In (1) and (2), P stands for probability. From (1), it is seen that

$$E(N) = \lambda, \qquad (3)$$

where E is the expected value operator. In other words, λ is the average number of exceedances per year.

For the magnitudes of the hazard events to be independent, x_o should be rather higher. In practice, it is selected in such a way that the number of exceedances over the length L of a record will be in the range from L to 5L. Once x_o has been each year from the record concerned, the second, third, etc... highest values in a particular year are disregarded even though they are higher than the annual maximum values in some other years. This disadvantage can be remedied by the use of the partial-duration series which comprises all values of the record exceeding some base value.

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Using this model, i.e. the two assumptions, one can establish several important results in the following sections.

Number of Exceedances with a Higher Base Value

Let $x > x_o$, and M be the number of exceedances with reference to x in a year. Then

$$M = \sum_{i=1}^{N} B_i, \tag{4}$$

where B_i is the Bernoulli random variable associated with the ith hazard event. In other words:

$$B_i = \begin{matrix} 1 \text{ if } X > x \\ 0 \text{ if } X \leq x, \end{matrix}$$

These Bernoulli variables are independent and identically distributed with the following probability mass:

$$g(b) = \begin{matrix} P[X > x] = 1-F(x) \text{ for } b = 1, \\ P[X \leq x] = F(x) \quad \text{ for } b = 0, \\ 0, \text{ otherwise.} \end{matrix} \tag{5}$$

From the result obtained by Shane and Lynn[3], one can write:

$$P[M = m] = \sum_{n=m}^{\infty} P[N = n] \, [g(m)]^{n*}, \tag{6}$$

in which $[g(m)]^{n*}$ is the nth convolution of g(m) with itself. One has:

$$[g(m)]^{n*} = \binom{n}{m} [1 - F(x)]^m \, [F(x)]^{n-m}, \tag{7}$$

where

$$\binom{n}{m} = \frac{n!}{m!\,(n-m)!}.$$

Using (1) and (7), one obtains

$$P[M = m] = \exp(-\lambda) \, \frac{\{\lambda[1 - F(x)]\}^m}{m!} \sum_{n=m}^{\infty} \frac{[\lambda F(x)]^{n-m}}{(n-m)!},$$

which can be reduced to:

$$P[M = m] = \frac{\{\lambda[1 - F(x)]\}^m}{m!} \exp\left\{-\lambda[1 - F(x)]\right\}. \tag{8}$$

So the number of exceedances with reference to any base value $x > x_o$ also follows the Poisson distribution. In this case the exceedance rate is:

$$E(M) = \lambda', \tag{9}$$

where

$$\lambda' = \lambda[1 - F(x)], \tag{10}$$

which clearly depends upon λ and F(x).

Largest Exceedances

Due to assumption (ii), the largest exceedances in different years are independent and identically distributed. Let χ be the random variable which has the common distribution of these largest exceedances. Then the distribution function of χ can be written as follows:

$$H(x) = P[\chi \leq x] = \sum_{n=0}^{\infty} P[N = n] \left\{P[\chi \leq x]\right\}^n, \tag{11}$$

Since χ is among the hazard magnitudes,

$$P[\chi \leq x] = F(x). \tag{12}$$

Substituting (1) and (12) into (11) gives:

$$H(x) = \exp(-\lambda) \sum_{n=0}^{\infty} [\lambda F(x)]^n/n!,$$

which reduces to:

$$H(x) = \exp\left\{-\lambda[1 - F(x)]\right\}. \tag{13}$$

This result is very important in establishing the relationship between the return periods T and T_A which are based upon the partial-duration series and the annual maxima series, respectively.

Relationship between T and T_A

The return period (also called the recurrence interval) of an event of a given magnitude is the average length of time between successive occurrences of that event. Now let x be a given magnitude with $x > x_o$. Since the mean number of exceedances per year is λ', that in T years is $\lambda'T$. When this is equal to 1, the corresponding value of x has a return period T. Therefore, one can write:

$$T = 1/\lambda', \tag{14}$$

or in view of (10):

$$T = \left\{\lambda[1-F(x)]\right\}^{-1}. \tag{15}$$

This return period is based upon the number of exceedances with reference to x. In other words, it is based upon the partial-duration series.

Now let T_A denote the return period of magnitude x corresponding to the annual maxima series. In statistical analysis, this series is treated as consisting of independent and identical random variables. This treatment is also adopted here, so that the maximum value in one year has the same distribution as the maximum value in another year. Since the largest exceedance in a year when it exists must be the maximum value of that year, the above treatment leads to the conclusion that the distribution of the largest exceedances obtained in (13) is also the distribution of the annual maxima. Hence the return period T_A can be obtained from its usual definition as:

$$T_A = 1/P[\chi > x] = [1 - H(x)]^{-1},$$

In view of (10) and (13), one can write:

$$T_A = [1 - \exp(-\lambda')]^{-1},$$

which after using (14) becomes:

$$T_A = [1 - \exp(-\frac{1}{T})]^{-1}, \qquad (16)$$

or

$$T = [\ln T_A - \ln(T_A - 1)]^{-1}, \qquad (17)$$

This result was obtained by Chow[4] using an approximate method. Using this result, one can easily verify that the two return periods are significantly different only at low values (less than 5 years).

New Approach to the Gumbel Distribution

The extreme-value type-1 distribution commonly referred to as the Gumbel distribution, is most widely used in fitting annual maxima series. For those series in Thailand and Laos, its applicability has been proven[5,6,7]. So far, there have been two approaches, one by Gumbel[8], the other be Jowitt[9] to arriving at this distribution. In this study, the Gumbel distribution is shown to be obtained by a suitable choice of the distribution used for the magnitude of the hazard events and by an appropriate selection of the base value x_0.

First, in view of (3), the distribution of largest exceedances can be rewritten as:

$$H(x) = \exp\{- E(N) [1 - F(x)]\} \qquad (18)$$

Now, assuming that F(x) is the shifted exponential distribution, i.e.

$$F(x) = 1 - \exp[- (x - x_0)/\beta], \qquad (19)$$

where β is the paramater, and that the exceedance rate is equal to unity:

$$E(N) = \lambda = 1, \qquad (20)$$

then (18) becomes:

$$H(x) = \exp\{- \exp[-(x - x_0)/\beta]\}, \qquad (21)$$

whis is the Gumbel distribution with x_0 and β beïng the location and concentration parameters, respectively. This result shows that the Gumbel distribution is the distribution of the maximum exceedances in different years when the base value x_0 is determined so that the two conditions expressed by (19) and (20) are both satisfied.

Application to Actual Data

In the foregoing theoretical analysis, serveral assumptions were made. Of course, the derived formulae hold only when these assumptions are valid. Like any other mathematical model which aims at solving some practical problem, the present model is not always applicable. However, for the rainfall in the Northeast, it is realistic and very satisfactory, as it can be seen from the analysis of some typical sets of data listed in Table 1. To start with, the two basic assumptions must be verified.

TABLE 1: LIST OF RAINFALL STATIONS EMPLOYED

Station	Location	Period of record used
S1	Chum Phae. Khon Kaen	1952 – 1976
S2	RID office, Khon Kaen	1955 – 1977
S3	Huai Kha Khang, Maha Sarakham	1954 – 1977
S4	Nong Sun Agricultural Station, Nakon Ratchasima	1955 – 1977
S5	Muang, Nong Khai	1952 – 1977
S6	Muang, Roi Et	1952 – 1977
S7	Agricultural Experimental Station, Roi Et	1952 – 1977
S8	Amnat Charoen, Ubon Ratchathani	1952 – 1975
S9	Warin Chamrap, Ubon Ratchathani	1952 – 1977

On the Poisson Assumption

Normally, the base value x_o should be high enough. Once x_o has been selected, the validity of the Poisson assumption may be assessed using the resulting partial-duration series which consists of all daily rainfall depths exceeding x_o. Let n_i be the number of exceedances in successive years of the record, $i = 1, 2, \ldots, L$, then the assumption is tested using the following chi-square statistic:

$$\chi^2 = \sum_{j=1}^{K} (O_j - E_j)^2/E_j, \tag{22}$$

where O_j and E_j are respectively the observed and expected frequencies at point j. The value of K is selected as the maximum value of n_i:

$$K = \max\{n_i\}, \; i = 1, \ldots L. \tag{23}$$

Define

$$N_p = \sum_{i=1}^{L} n_i, \tag{24}$$

then the expected frequency E_j is computed by

$$E_j = N_p \, P[N = j] = N_p \, \lambda^j \exp(-\lambda)/j!. \tag{25}$$

Since λ has to be estimated, the above statistic has K-2 degrees of freedom.

The chi-square test may be used for any distribution. In the present situation, for the Poisson assumption, the Fisher dispersion test can also be used. This test makes use of the following statistic

$$d = \sum_{i=1}^{L} (n_i - \hat{\lambda})^2/\hat{\lambda}, \tag{26}$$

where $\hat{\lambda}$ is an estimator of λ. When $\hat{\lambda}$ is sufficiently large ($\lambda > 5$), d is approximately distributed as a chi-square variable with L-1 degrees of freedom. However, as indicated by Sukhatme[10], this approximation is still acceptable provided that $L > 5$ if $\hat{\lambda} > 1$ and $L > 15$ if $\hat{\lambda} < 1$.

For all the data sets under consideration, L is greater than 20, and the Fisher dispersion test may be readily applicable to all possible values of $\hat{\lambda}$. The results of such evaluation are summarized in Table 2 where x_o may vary from station to station. These clearly show that the assumption is valid.

TABLE 2: STATISTICAL EVALUATION OF THE POISSON ASSUMPTION

Station	x_o (mm)	χ^2			d		
		(1)	(2)	(3)	(1)	(2)	(3)
S1	62	8.367	4	9.488	32.909	24	36.415
S2	74	4.696	3	7.815	31.241	22	33.924
S3	70	0.320	1	3.841	17.696	23	35.172
S4	80	2.551	2	5.991	17.533	22	33.924
S5	84	4.006	2	5.491	18.333	21	32.671
S6	80	2.826	2	5 991	18.889	25	37.652
S7	80	4.131	3	7.815	31.047	25	37.652
S8	102	1.339	2	5.991	32.000	23	35.172
S9	82	5.478	3	7.815	22.588	25	37.652

Notes: (1) Computed Value
(2) Degree of freedom
(3) Critical value at 5% significance level.

On the Independence Assumption

The independence assumption for daily rainfall depths exceeding x_o is even more acceptable than in the case of daily streamflows (see Ref. 3). However, to provide a rather comprehensive analysis, this assumption is assessed by using the *turning point test*. In the observed series x_t, $t=1, \ldots, N_p$, a turning point occurs at $t=i$ if x_i is either greater than x_{i-1} and x_{i+1}, or less than these two adjacent values. The turning points in a sequence consisting of N_p values may be approximated by a normal distribution with mean μ and variance σ^2 given by[11]:

$$\mu = 2(N_p - 2)/3,$$
$$\sigma^2 = (16N_p - 29)/90.$$

Thus the randomness (or independence) assumption can readily be evaluated. Typical results are shown in Table 3. These result indicate how high x_o must be for it to be accepted. It should also be noted that the validity of the Poisson assumption does not have any implication on the randomness of the resulting partial-duration series.

TABLE 3: TYPICAL RESULTS OF A STATISTICAL EVALUATION OF THE INDEPENDENCE ASSUMPTION

Station	Computed Values		
S2	2.590*(70)	1.936 (72)	1.365 (74)
S4	2.452*(78)	1.942 (80)	1.743 (82)
S6	2.380*(78)	1.921 (80)	1.681 (82)
S8	2.110*(100)	1.678 (102)	1.546 (104)

Notes: In parentheses is selected value of x_o, case with (*) is rejected at 5% significance level.

On the Shifted Exponential Assumption

As previously indicated, the distribution to be used for the magnitudes of hazard events is determined from actual data. However, in order to show that the Gumbel distribution is indeed the distribution of largest exceedances subject to a constraint on the exceedance rate it has been assumed to be the shifted exponential distribution. To justify this assumption, the Kolmogorov-Smirnov test can be employed. The procedure is as follows:

1. Estimate the parameter β by

$$\hat{\beta} = \bar{x} - x_o, \tag{27}$$

where

$$\bar{x} = \frac{1}{N_p} \sum_{i=1}^{N_p} x_i \, , \; x_i > x_o \, . \tag{28}$$

2. Arrange the sequence $x_i, i=1, \ldots, N_p$ in ascending order $x_1 \leq x_2 \leq \ldots \leq x_N$, and assign the rank i to x_i.

3. Compute the following statistic:

$$\triangle = \max_{i = 1, \ldots, N_p} \left| F(x_i) - i/N_p \right| , \tag{29}$$

where $F(x_i)$ is the value of F at x_i, with $\hat{\beta}$ having been inserted into (19).

4. Compare this statistic with the tabulated critical value $\triangle_o$ at the selected significance level (in this study, it is equal to 5 per cent). If $\triangle < \triangle_o$, the assumption is accepted. The value of $\triangle_o$ is available in most statistical books as well as in Yevjevich[12] and Haan[13].

As summarized in Table 4, the magnitudes of hazard events can be fitted very well by the shifted exponential distribution.

TABLE 4: RESULTS OF FITTING THE MAGNITUDES OF HAZARD EVENTS BY THE SHIFTED EXPONENTIAL DISTRIBUTION

Station	S1	S2	S3	S4	S5	S6	S7	S8	S9
x_o	62	70	70	78	80	78	78	98	78
N_p	33	32	23	35	37	41	51	30	41
Δ	0.089	0.081	0.187	0.177	0.073	0.152	0.139	0.111	0.060
Δ_o^*	0.231	0.234	0.275	0.224	0.218	0.208	0.187	0.242	0.208
x_o	64	72	72	80	82	80	80	100	80
N_p	32	30	20	30	35	36	42	27	39
Δ	0.082	0.087	0.222	0.144	0.083	0.168	0.124	0.106	0.091
Δ_o^*	0.234	0.242	0.294	0.242	0.224	0.221	0.205	0.254	0.213
x_o	66	74	74	82	84	82	82	102	82
N_p	28	29	18	25	33	32	39	24	34
Δ	0.107	0.106	9.249	0.119	0.093	0.183	0.130	0.090	0.065
Δ_o^*	0.250	0.245	0.309	0.264	0.231	0.234	0.213	0.269	0.227

(*) at 5% significance level

Estimation of Exceedance Rate

In the verification of the Poisson assumption, λ must be estimated from the number of hazard events. This can easily be obtained by taking

$$\hat{\lambda} = N_p/L, \tag{30}$$

where N_p is the total number of exceedances and L is the length of record in years as previously defined. When a base value x_o is selected, $\hat{\lambda}$ is estimated by using (30). However, for any new base value $x > x_o$, the corresponding exceedance rate should be computed by using (10):

$$\lambda' = \lambda[1 - F(x)].$$

However, if this was not known, and (30) was used instead (with N_p being the corresponding total number of exceedances), the resulting value for λ' may be quite different as shown in Table 5.

Notes: (1) In Table 2, the value of x_o has been selected so that the Poisson, independence and shifted exponential assumptions are all satisfied.

(2) From the results shown in Tables 2–4, it can be seen that at a given station, the shifted exponential assumption is justified even for relatively small values of x_o. The independence assumption is most difficult to justify.

TABLE 5: DIFFERENCE BETWEEN ESTIMATED AND THEORETICAL VALUES OF λ' (STATION S1)

x_o	64	66	68	70
eatimated	1.280	1.120	1.080	1.000
theoretical	1.065	0.850	0.630	0.632

Remark on the Gumbel Distribution

Since in this study, the magnitudes of hazard events have been shown to be independent and fitted by the shifted exponential distribution, one should expect that the Gumbel distribution can satisfactorily represent the maximum exceedances in different years of records when λ is 1. However, λ is only determined after a base value has been selected. Therefore, λ is obtained from its estimator shown in (30). By varying x_o, one may expect that when λ is close to 1, fitting the sequence of maximum exceedances by the Gumbel distribution, with predetermined values for the parameters x_o and β is acceptable. The results shown in Table 6 clearly support this idea, and thus the situation can actually be realized.

TABLE 6: TYPICAL RESULTS OF FITTING THE SEQUENCE OF MAXIMUM EXCEEDANCES BY THE GUMBEL DISTRIBUTION (EVALUATED BY THE KOLMOGOROV-SMIRNOV TEST)

Station	(1)	(2)	(3)	(1)	(2)	(3)
S1	1.080	0.210	0.318	1.00	0.208	0.327
S2	1.087	0.231	0.361	0.913	0.242	0.375
S4	1.087	0.320	0.337	0.957	0.312	0.349
S5	1.045	0.294	0.349	0.955	0.310	0.369

Notes : (1) Value of λ
(2) Value of the Kolmogorov-Smirnov test ($\triangle$)
(3) Critical value at 5% significance level

Remark on Relationship between T and T_A

The relationship between T and T_A as expressed by (17) was obtained by arguing that the largest exceedances and the annual maxima have the same distribution. In most situations, one actually has two samples, one for the maximum exceedances and the other for the annual maxima. Obviously the former sample is a part of the latter. They are identical only when the base value x_o is selected so that the number of exceedances in any year of the record is not zero. To justify that they have the same distribution, the simplest way is to verify that the annual maxima also fit the distribution derived in (13) for the maximum exceedances. The computed values of the Kolmogorov-Smirnov test (Table 7) assert such a fitting.

TABLE 7 FITTING THE ANNUAL MAXIMA SEQUENCE BY THE DISTRIBUTION OF MAXIMUM EXCEEDANCES

Station	x_o	Δ	Δ_o
S1	62	0.097	0.264
S2	74	0.264	0.274
S3	70	0.157	0.269
S4	80	0.137	0.275
S5	84	0.088	0.281
S6	80	0.099	0.259
S7	80	0.115	0.259
S8	102	0.126	0.269
S9	82	0.079	0.259

Summary and Conclusions

In this study, a mathematical model which consists of a Poisson distribution for the number of exceedances and a shifted exponential distribution for the magnitudes of the hazard events has been proposed for the analysis of the partial-duration series of (exceeding) depths of daily rainfall in the Northeast of Thailand. With this model, an important relationship between the return periods which are based respectively upon the partial-duration and annual maxima series is analytically derived. More-ever, the Gumbel distribution is shown to be obtained as the distribution of the annual largest exceedances when the number of exceedance is 1 on the average.

All the assumptions involved have been justified using suitable statistical tests and actual rainfall data, and thus the model is readily applicable to the investigation of the maximum depths of daily rainfall in the region.

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บทคัดย่อ

บทความนี้เสนอวิธีการใช้แบบจำลองคณิตศาสตร์ เพื่อการวิเคราะห์ปริมาณฝนที่มากที่สุดประจำวันสำหรับภาคอิสานของประเทศไทย แบบจำลองนี้ใช้วิธีการกระจายแบบปัวซอง สำหรับความน่าจะเป็นของจำนวนฝนที่ตกต่อเนื่องกันในแต่ละปี และใช้การกระจายเอกซโพเนนเซียลแบบเลื่อน สำหรับหาปริมาณน้ำฝนที่ตกต่อวัน บทความนี้แสดงการพิสูจน์แบบจำลองโดยวิธีสถิติ และแสดงความสัมพันธ์ระหว่างจำนวนปีที่จะเกิดมีฝนมากที่สุดในลักษณะหนึ่ง เทียบกับอีกลักษณะหนึ่ง นอกจากนี้ยังแสดงว่า ถ้าโดยเฉลี่ยทั้งปีมีจำนวนวันซึ่งมีฝนตกมากกว่าค่าที่กำหนดให้เป็นหนึ่งวันแล้ว การกระจายของปริมาณฝนสูงสุดจะเป็นแบบกัมเบล